UNIVERSITY OF Southampton
School of Physics and Astronomy

Classical Mechanics

PHYS 2006

Tim Freegarde

Classical Mechanics

LINEAR MOTION OF SYSTEMS OF PARTICLES	centre of mass
	Newton's 2nd law for bodies (internal forces cancel)
	rocket motion
ANGULAR MOTION	rotations and infinitessimal rotations
	angular velocity vector, angular momentum, torque
	parallel and perpendicular axis theorems
	rigid body rotation, moment of inertia, precession
GRAVITATION & KEPLER'S LAWS	conservative forces, law of universal gravitation
	2-body problem, reduced mass
	planetary orbits, Kepler's laws
	energy, effective potential
NON-INERTIAL REFERENCE FRAMES	centrifugal and Coriolis terms
	Foucault's pendulum, weather patterns
NORMAL MODES	coupled oscillators, normal modes
	boundary conditions, Eigenfrequencies

Newton's law of Universal Gravitation

- Exact analogy of Coulomb electrostatic interaction
- gravitational force between two masses m_1 and m_2

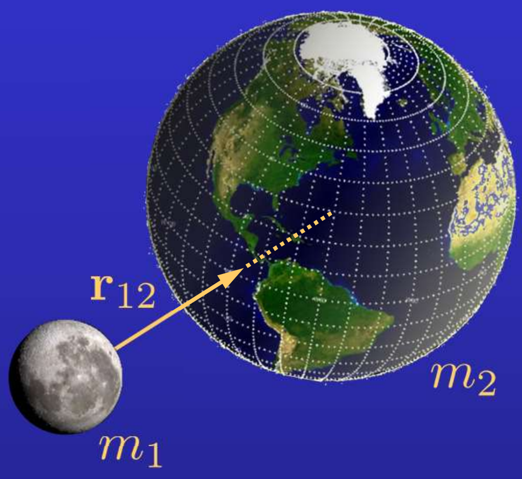
$$\mathbf{F}_{12} = -\mathbf{F}_{21} = \frac{Gm_1m_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = m_1\mathbf{g}$$

- gravitational field

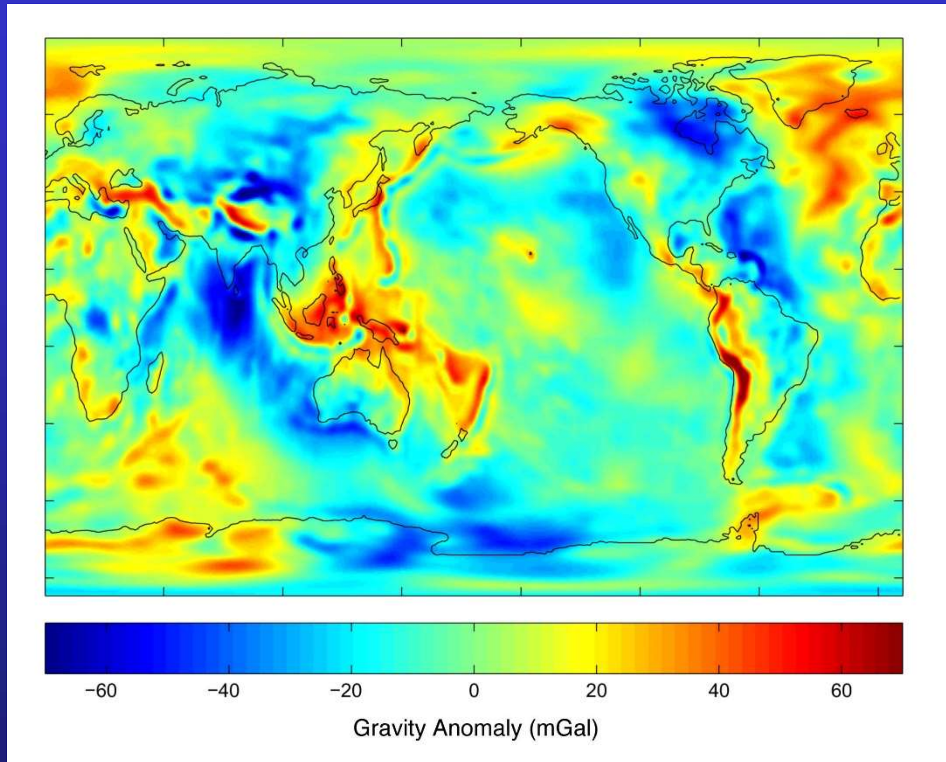
$$\mathbf{g}(\mathbf{r}_{12}) = -\frac{Gm_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = -\nabla\Phi$$

- gravitational potential

$$\Phi(r_{12}) = -\frac{Gm_1}{r_{12}}$$



Gravity anomaly



NASA/JPL/University of Texas Center for Space Research

- gravity anomaly = variation from uniform solid
- $1\,\mathrm{Gal} \equiv 0.01\,\mathrm{m\,s^{-1}} \approx g/1000$

Gravitational attraction of a spherical shell

- Exact analogy of Coulomb interaction
- gravitational force between two masses

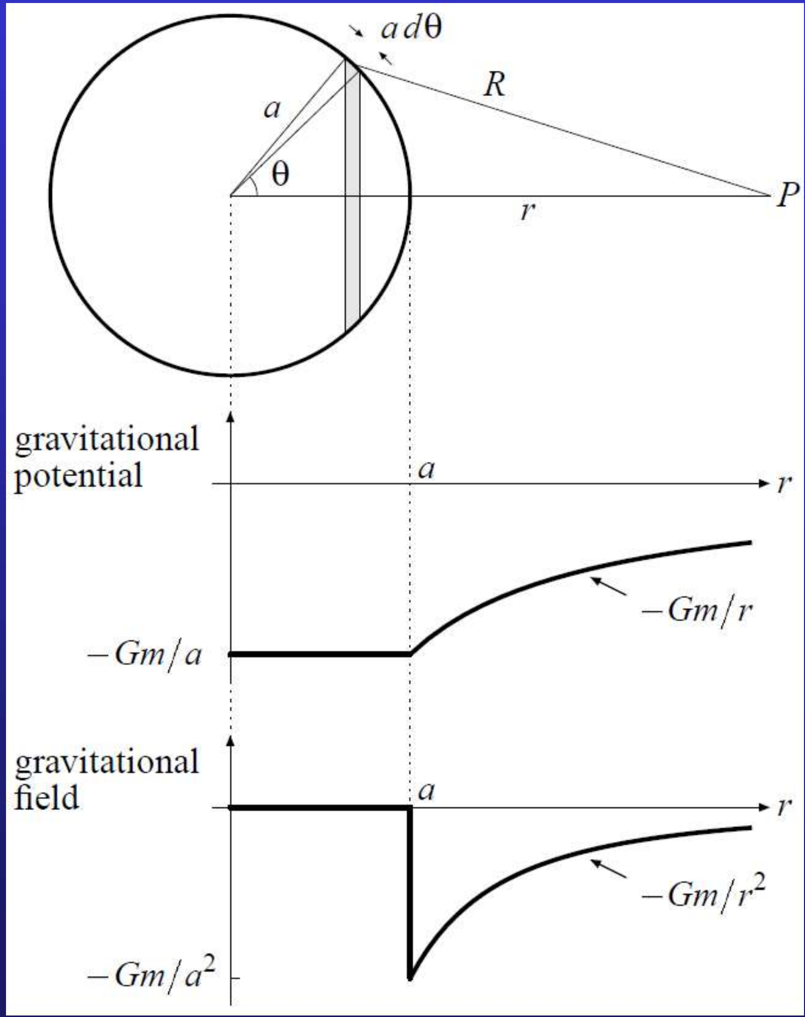
$$\mathbf{F}_{12} = -\mathbf{F}_{21} = \frac{Gm_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

- gravitational field

$$\mathbf{g}(\mathbf{r}_{12}) = -\frac{Gm_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

- gravitational potential

$$\Phi(r_{12}) = -\frac{Gm_1}{r_{12}}$$



Galilean equivalence principle

Apollo 15, David R Scott (7 August 1971)
www.youtube.com/watch?v=MJyUDpm9Kvk
history.nasa.gov/alsj/a15/a15.clsout3.html
nssdc.gsfc.nasa.gov/planetary/lunar/apollo_15_feather_drop.html

- gravitational field

$$\mathbf{g}(\mathbf{r}_{12}) = -\frac{Gm_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

- gravitational motion of mass m

$$m\ddot{\mathbf{r}} = m\mathbf{g} = -m\frac{GM}{r^2}\hat{\mathbf{r}}$$

inertial mass   gravitational mass

- equivalence principle

  the trajectory of a point-like mass in a gravitational field is independent of the composition and structure of the mass

$\Rightarrow$ inertial mass = gravitational mass

Newton’s law of Universal Gravitation

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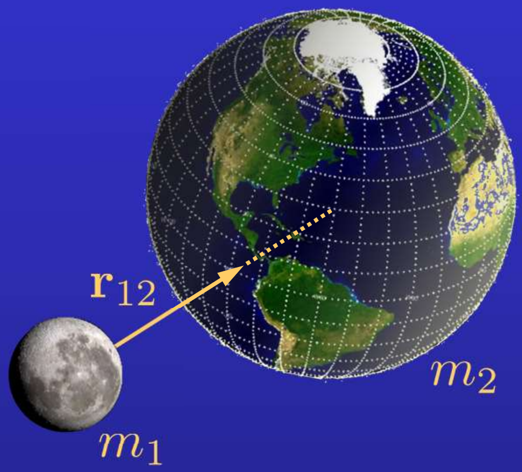
$$\mathbf{F}_{12} = -\mathbf{F}_{21} = \frac{Gm_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = m_1 \mathbf{g}$$



- gravitational field

$$\mathbf{g}(\mathbf{r}_{12}) = -\frac{Gm_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = -\boldsymbol{\nabla}\Phi$$

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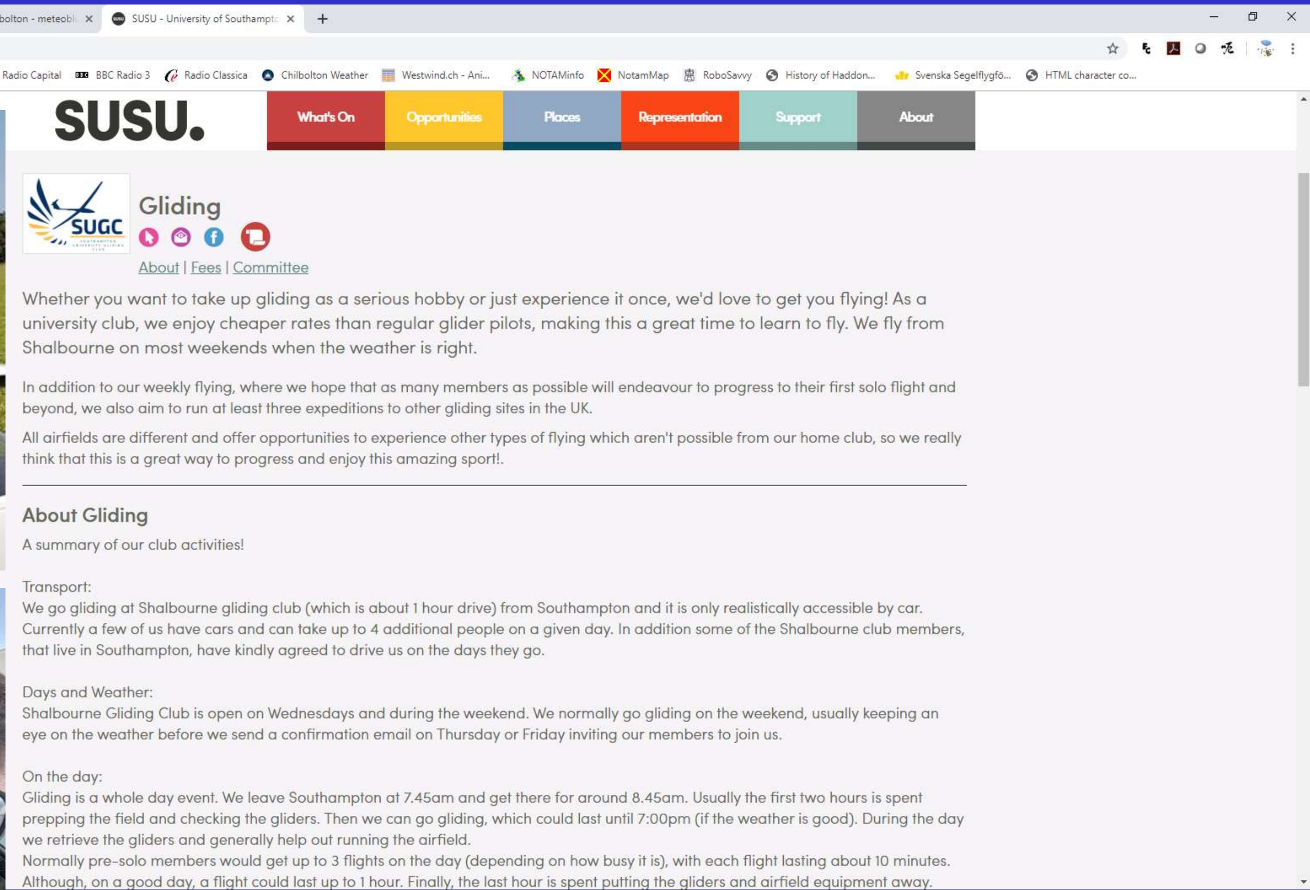
$$\Phi(r_{12}) = -\frac{Gm_1}{r_{12}}$$

GliderFX

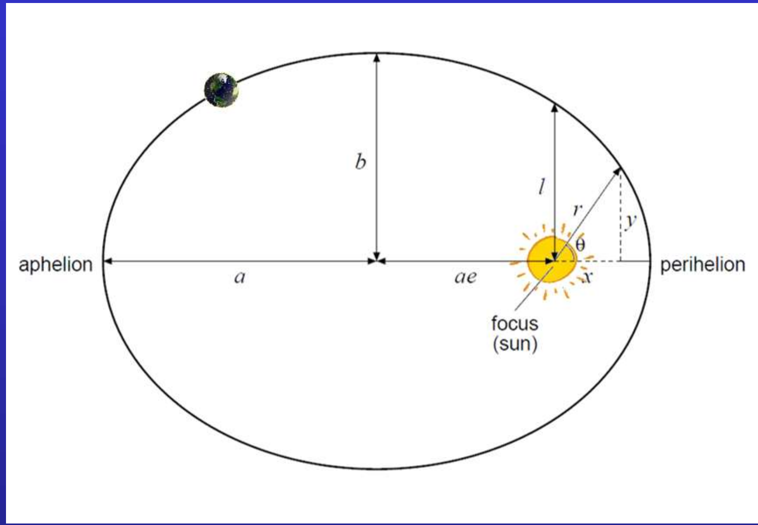
Project H2O	https://www.youtube.com/watch?v=VN_VG07qcPU	2:31
Project Gravity	https://www.youtube.com/watch?v=9G_djIzd21A	1:47

SU Gliding Club



gliding@soton.ac.uk

Elliptical orbit



- eccentricity e
- constant $k = GMm$
- semi latus rectum $l = \dfrac{L^2}{mk}$
- polar equation $\dfrac{l}{r} = 1 + e\cos\vartheta$
- Cartesian equation $\dfrac{(x+ae)^2}{a^2} + \dfrac{y^2}{b^2} = 1$
- semimajor axis $a = \dfrac{l}{1-e^2} = -\dfrac{k}{2E}$
- semiminor axis $b = \dfrac{l}{\sqrt{1-e^2}} = \sqrt{al}$
- total energy $E = -\dfrac{mk^2}{2L^2}\left(1-e^2\right) = -\dfrac{k}{2a}$

Conic section orbits

- eccentricity e
- constant $k = GMm$
- semi latus rectum $l = \frac{L^2}{mk}$
- polar equation $\frac{l}{r} = 1 + e\cos\vartheta$
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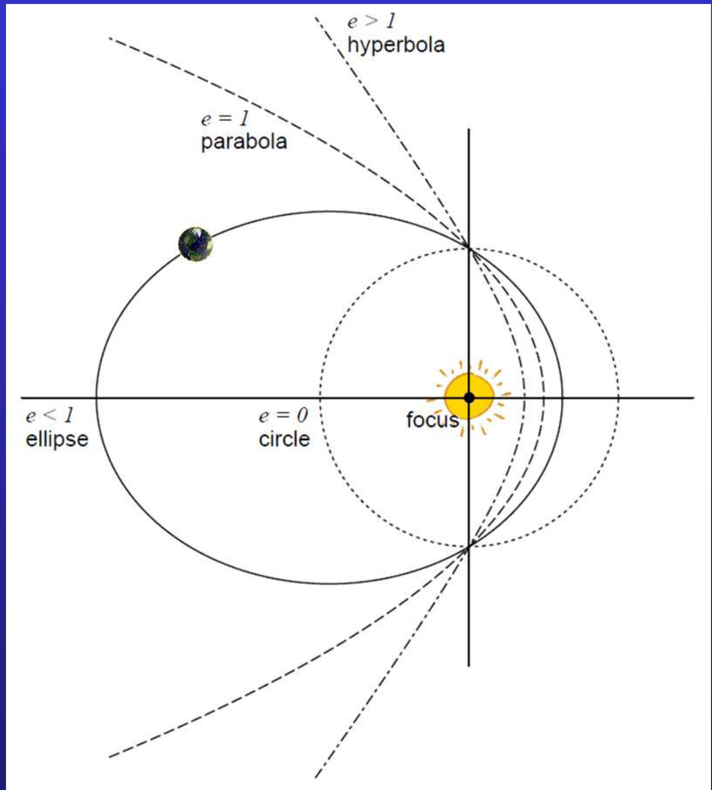
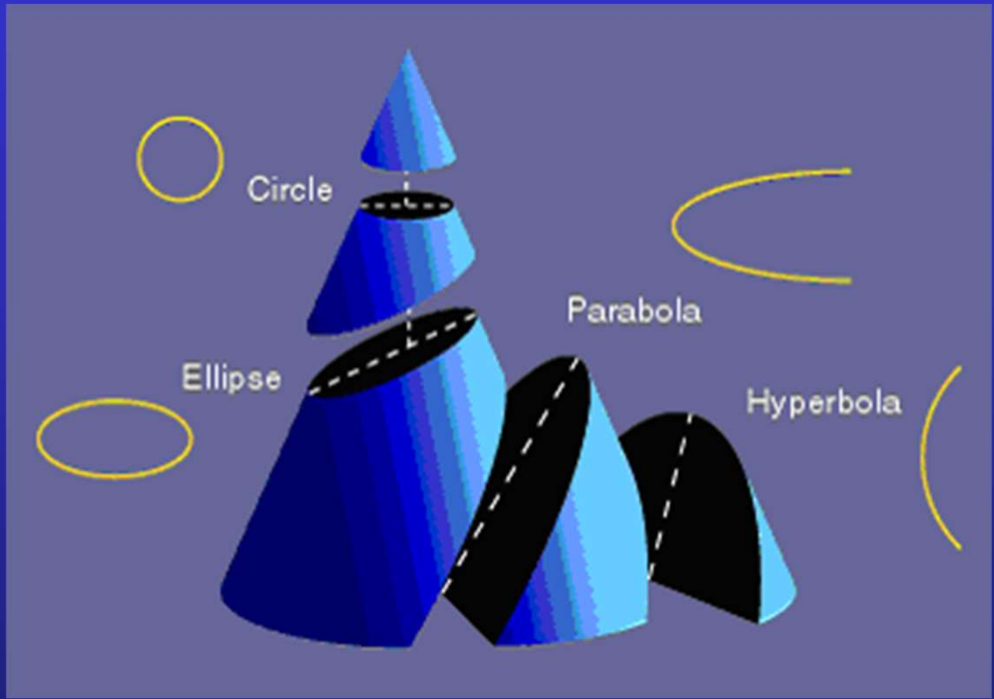
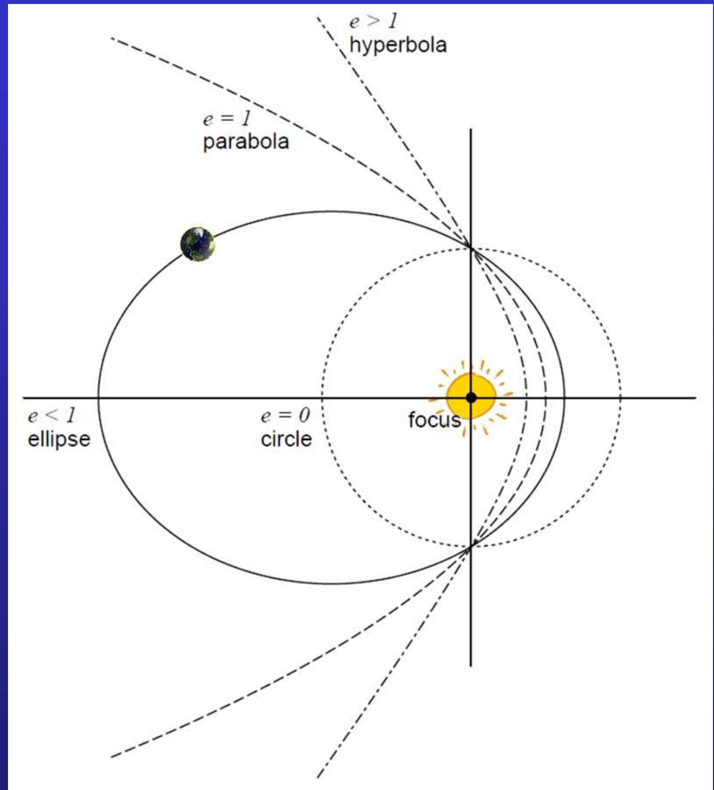


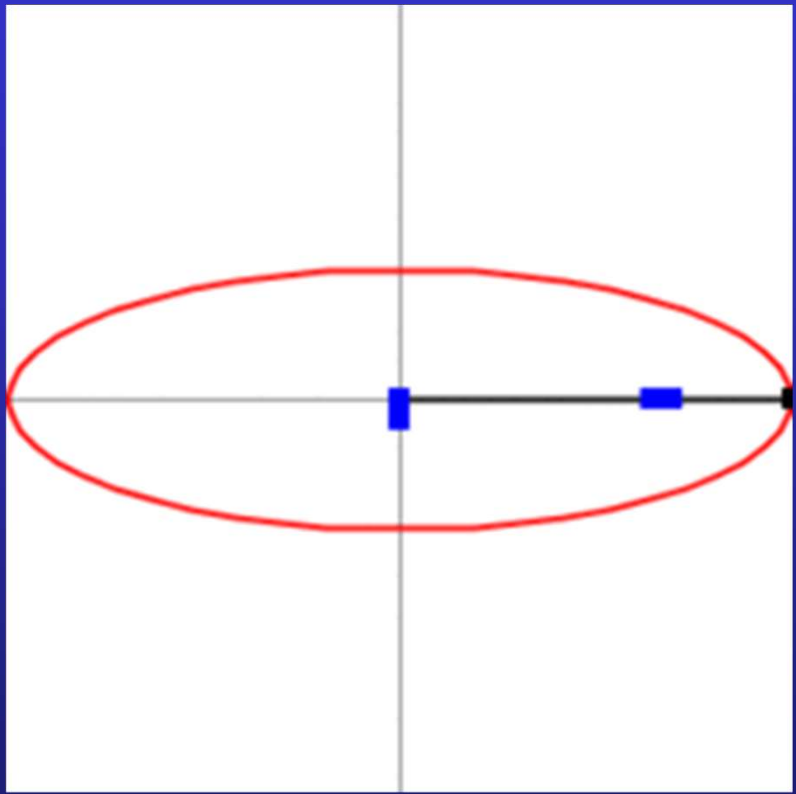
Conic section orbits



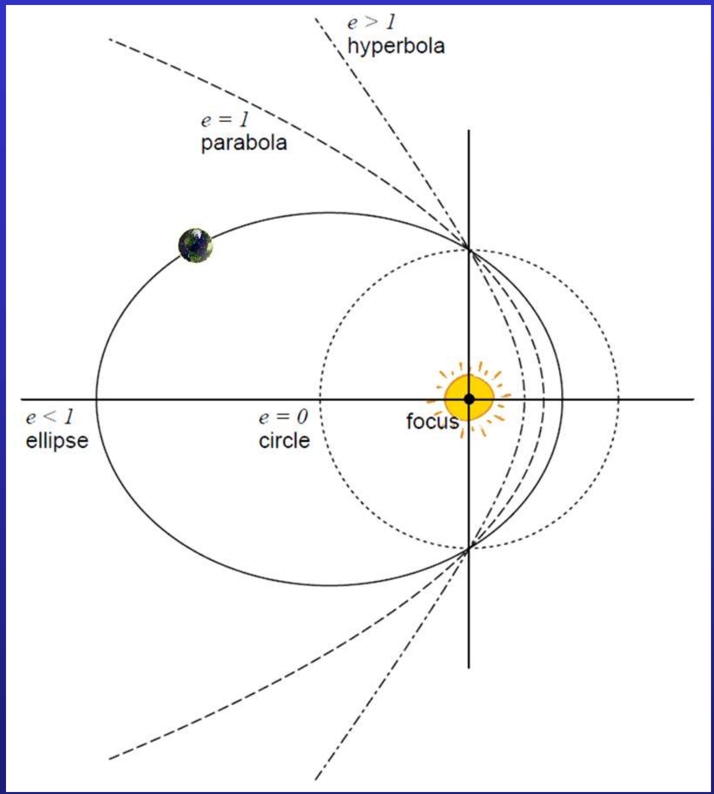
http://www.personal.psu.edu/smh408/



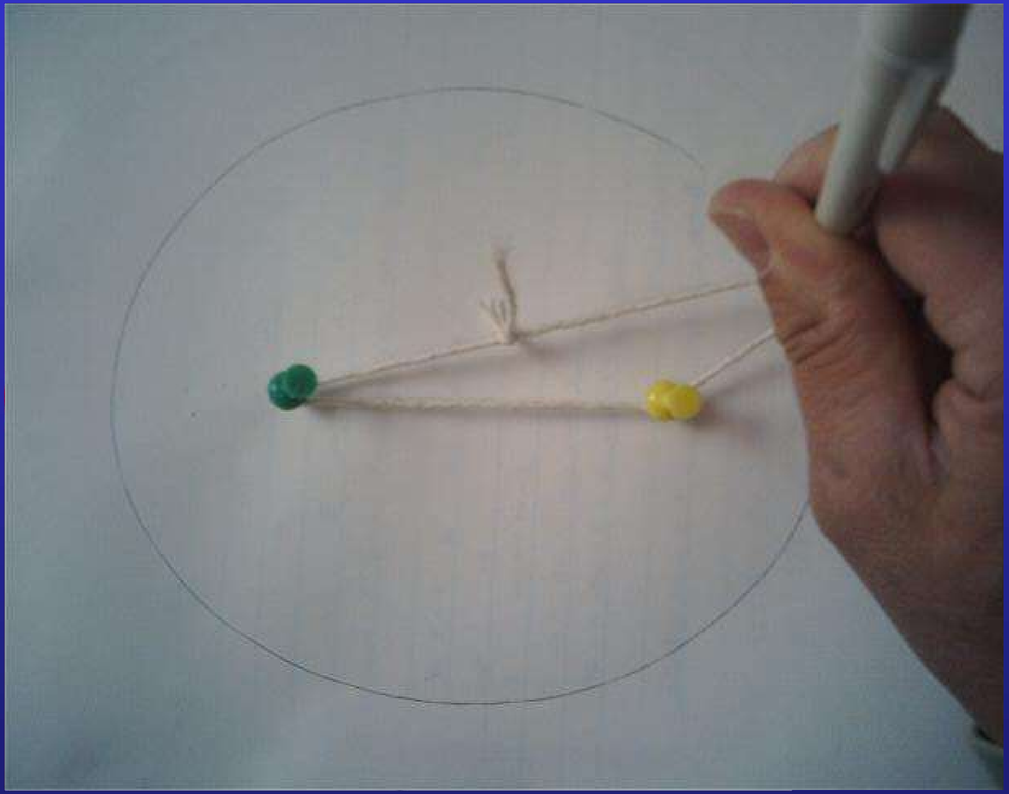
Conic section orbits



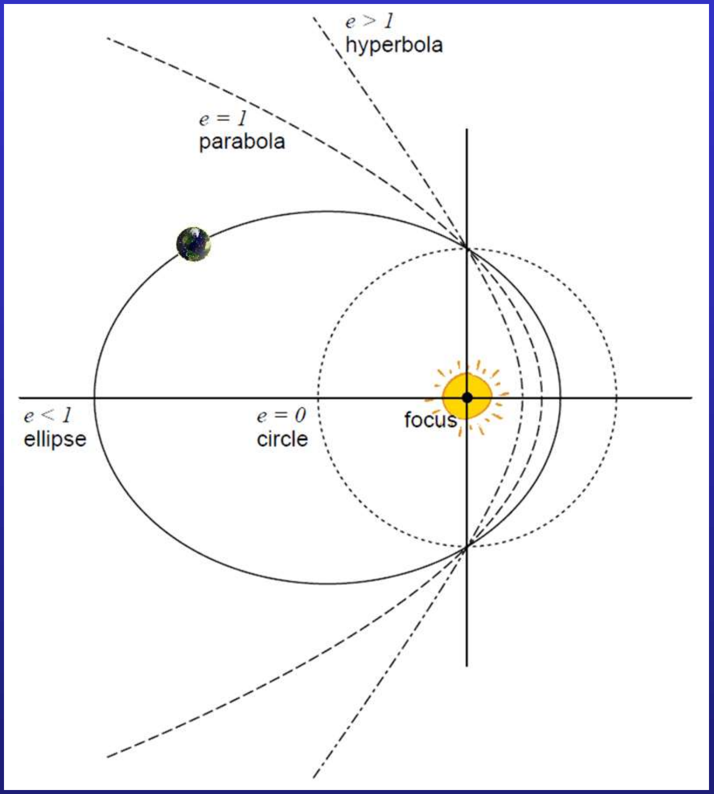
Alastair Rae



Conic section orbits



Magnus Manske



Kepler's laws

1. Planetary orbits are ellipses with the Sun at one focus

2. The radius vector from Sun to planet sweeps out equal areas in equal times

3. The square of the orbital period is proportional to the cube of the semimajor axis

$$T^2 \propto a^3$$