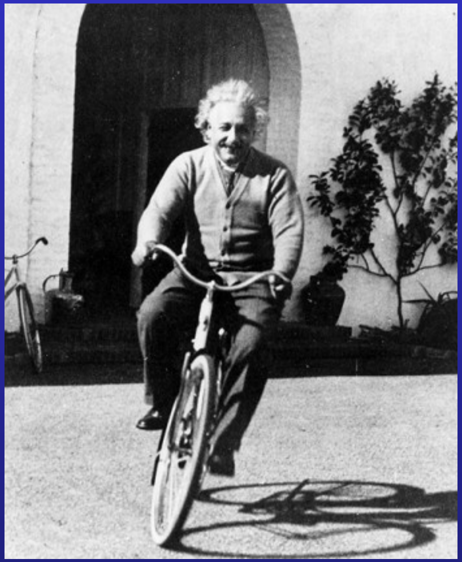
UNIVERSITY OF
Southampton
School of Physics
and Astronomy

Classical Mechanics

PHYS 2006

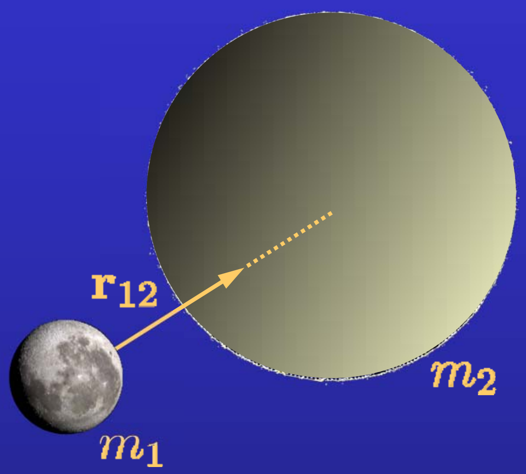
Tim Freegarde



Newton's law of Universal Gravitation

- Exact analogy of Coulomb electrostatic interaction
- gravitational force between two masses m_1 and m_2

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = \frac{Gm_1m_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = m_1\mathbf{g}$$



- gravitational field

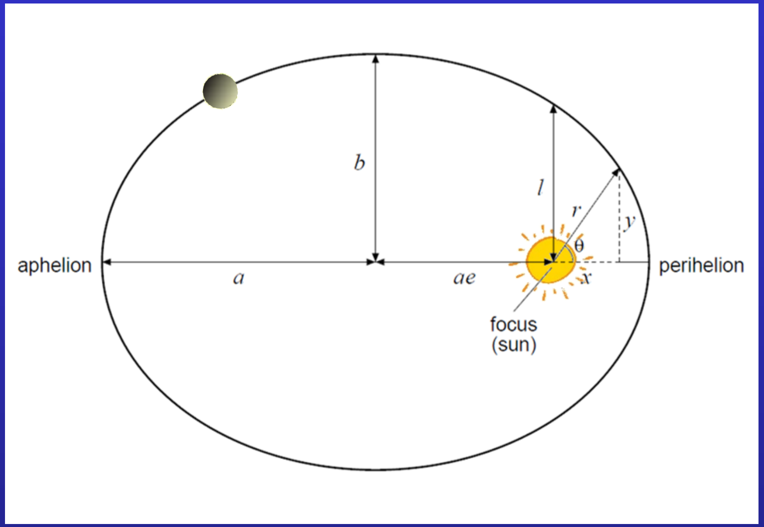
$$\mathbf{g}(\mathbf{r}_{12}) = -\frac{Gm_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = -\boldsymbol{\nabla}\Phi$$

- gravitational potential

$$\Phi(r_{12}) = -\frac{Gm_1}{r_{12}}$$

Elliptical orbit

- eccentricity e
- constant $k = GMm$
- semi latus rectum $l = \dfrac{L^2}{mk}$
- polar equation $\dfrac{l}{r} = 1 + e\cos\vartheta$
- Cartesian equation $\dfrac{(x+ae)^2}{a^2} + \dfrac{y^2}{b^2} = 1$
- semimajor axis $a = \dfrac{l}{1-e^2} = -\dfrac{k}{2E}$
- semiminor axis $b = \dfrac{l}{\sqrt{1-e^2}} = \sqrt{al}$
- total energy $E = -\dfrac{mk^2}{2L^2}\left(1-e^2\right) = -\dfrac{k}{2a}$



Conic section orbits

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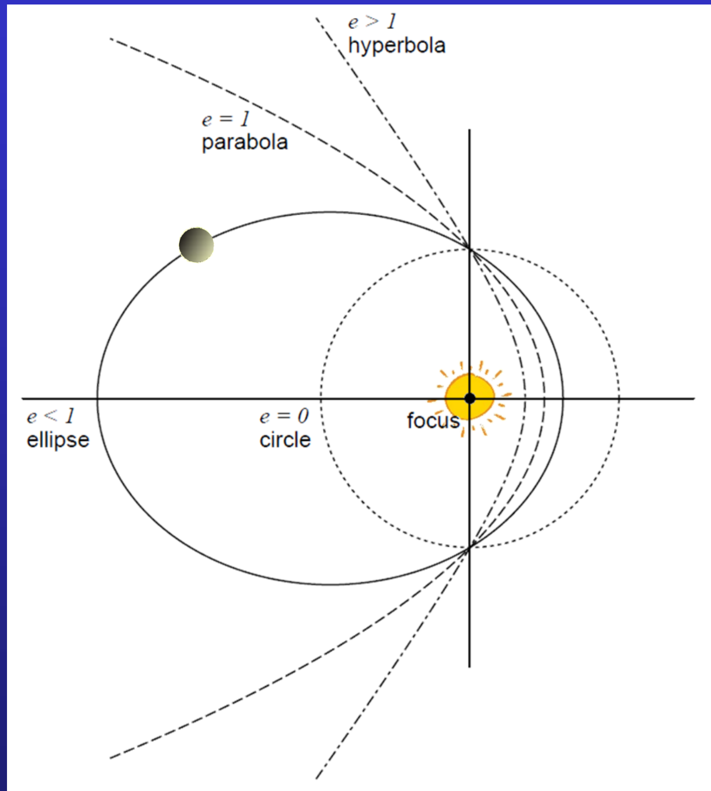
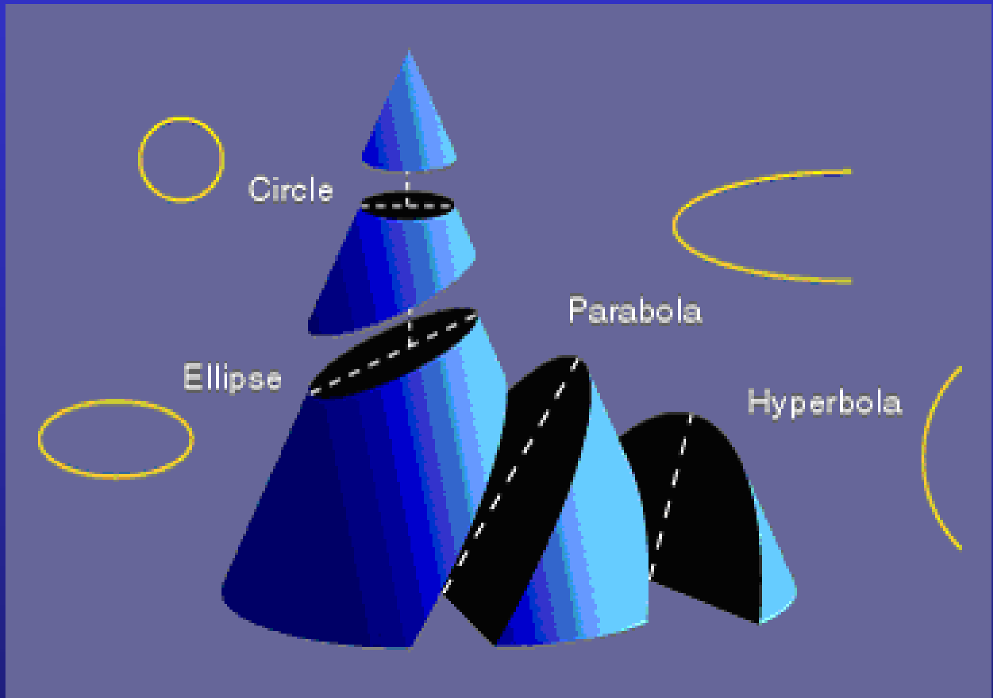
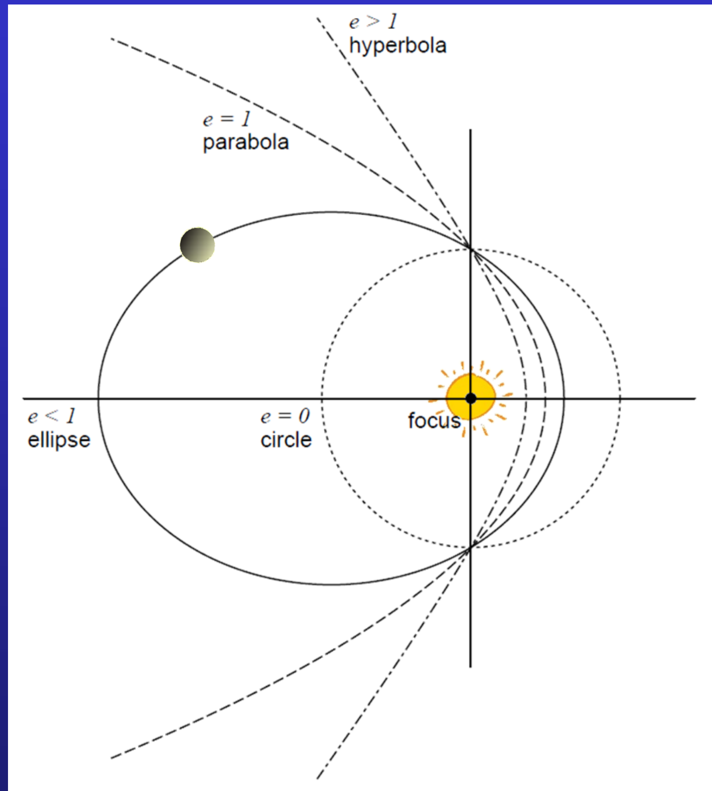


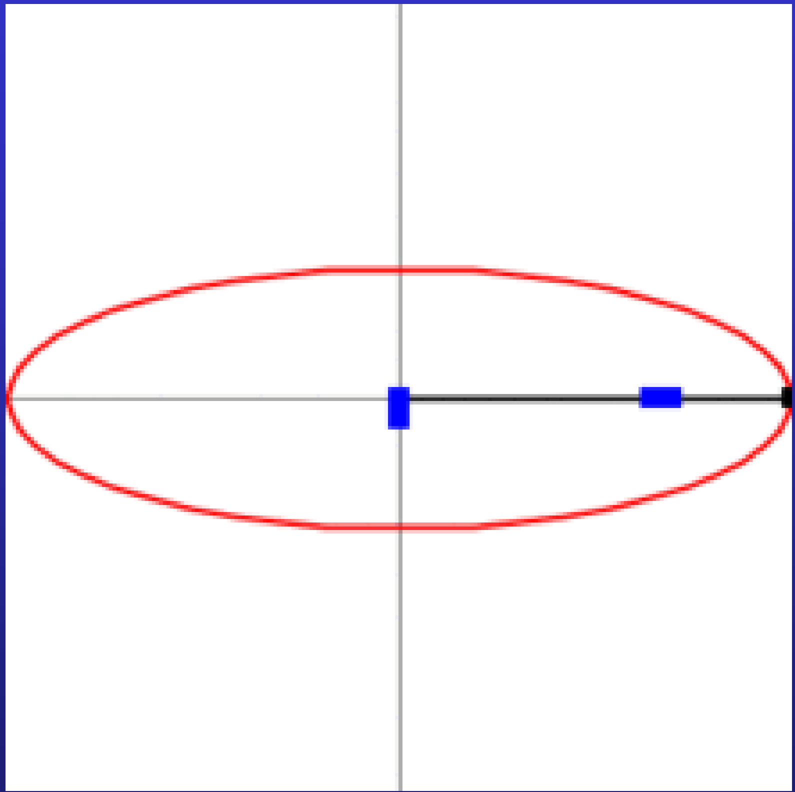
Conic section orbits



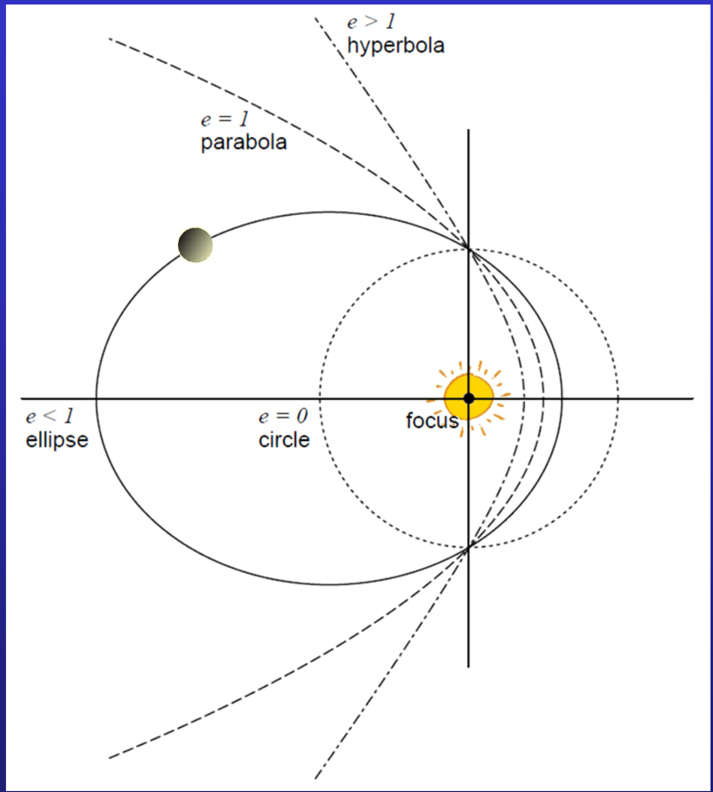
http://www.personal.psu.edu/smh408/



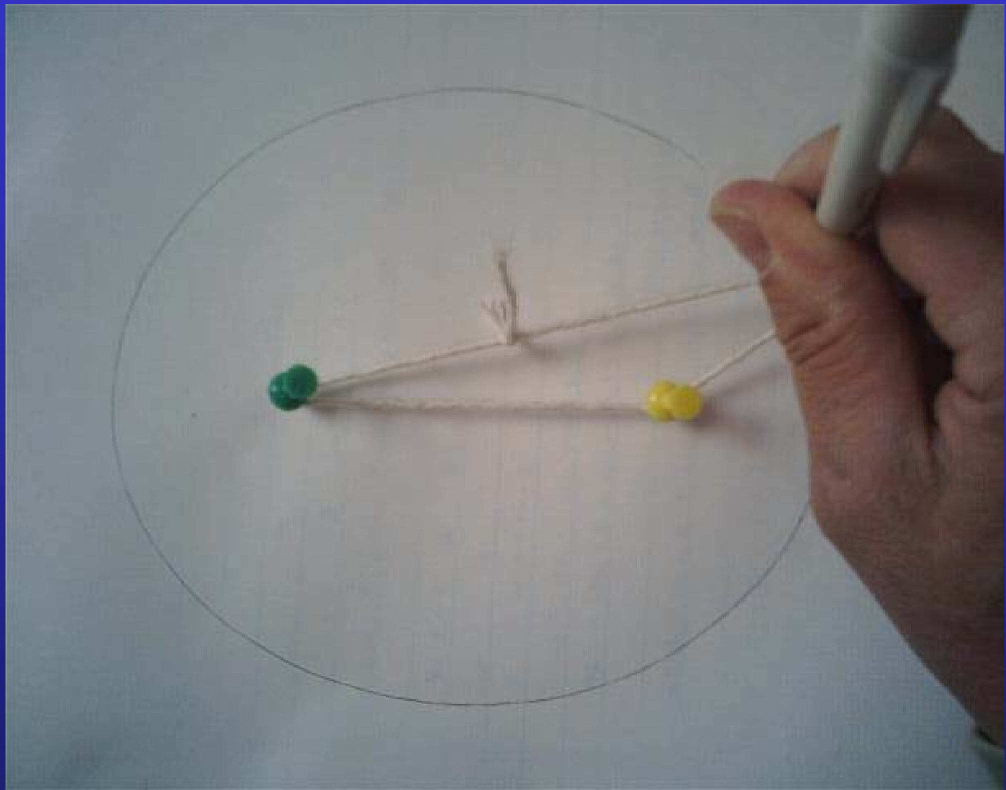
Conic section orbits



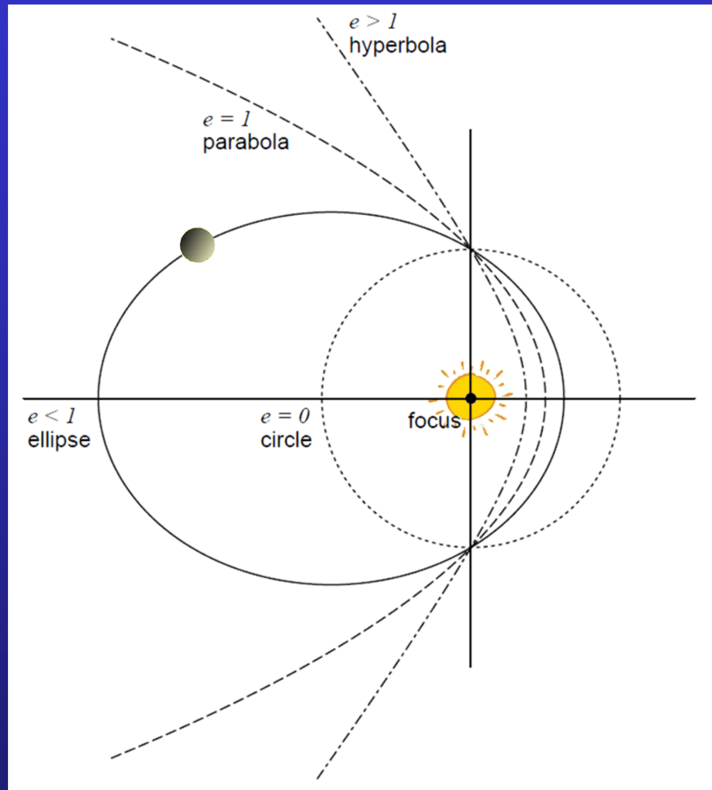
Alastair Rae



Conic section orbits



Magnus Manske



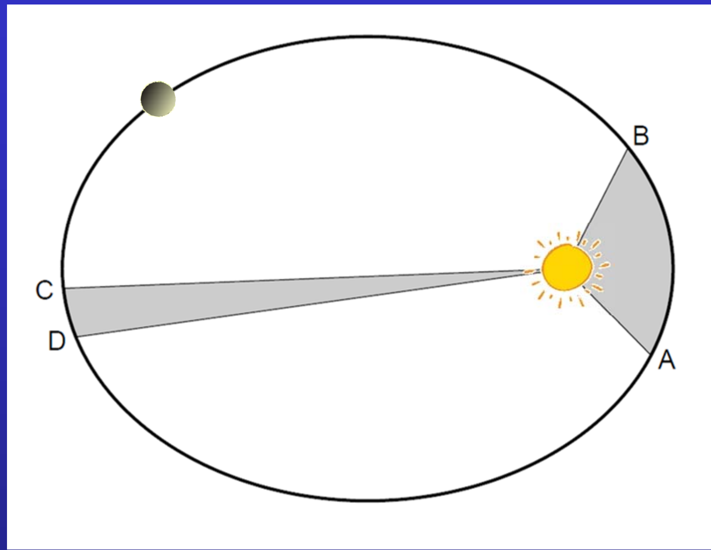
Kepler's laws

1. Planetary orbits are ellipses with the Sun at one focus

2. The radius vector from Sun to planet sweeps out equal areas in equal times

3. The square of the orbital period is proportional to the cube of the semimajor axis

$$T^2 \propto a^3$$



Conic section orbits

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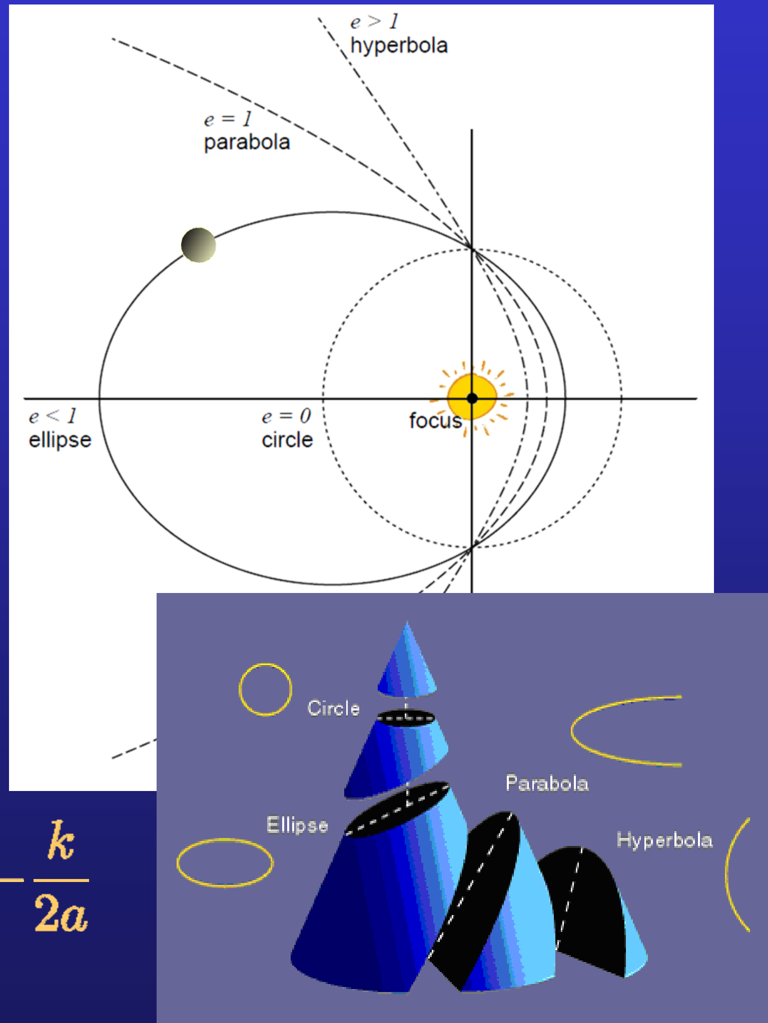

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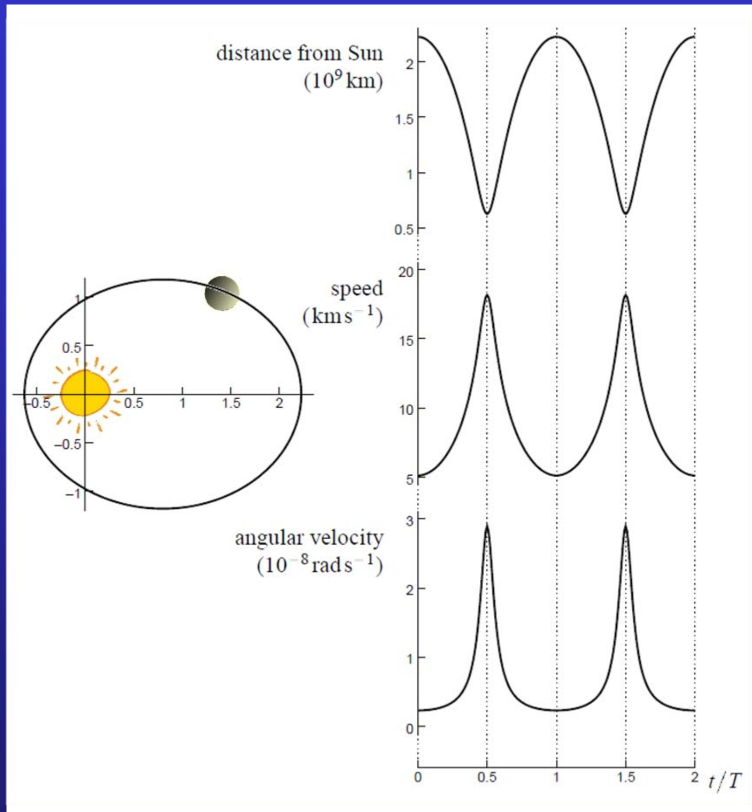


Effective potential

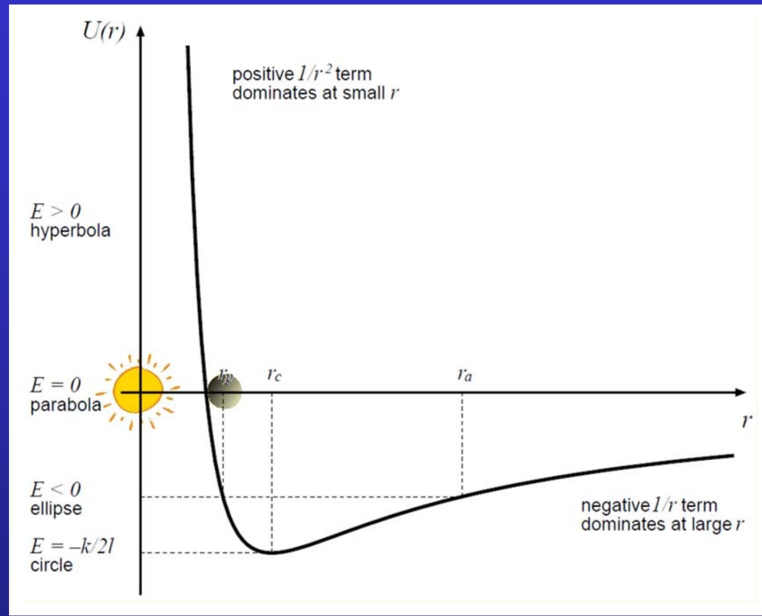
- angular momentum conserved

$$E = \frac{1}{2} m \dot{r}^2 + U_L(r)$$

where

$$U_L(r) = \frac{L^2}{2m} \frac{1}{r^2} - \frac{GM}{r}$$

- effective potential for radial motion



Yukawa potential

HIDEKI YUKAWA
(1907-1981)

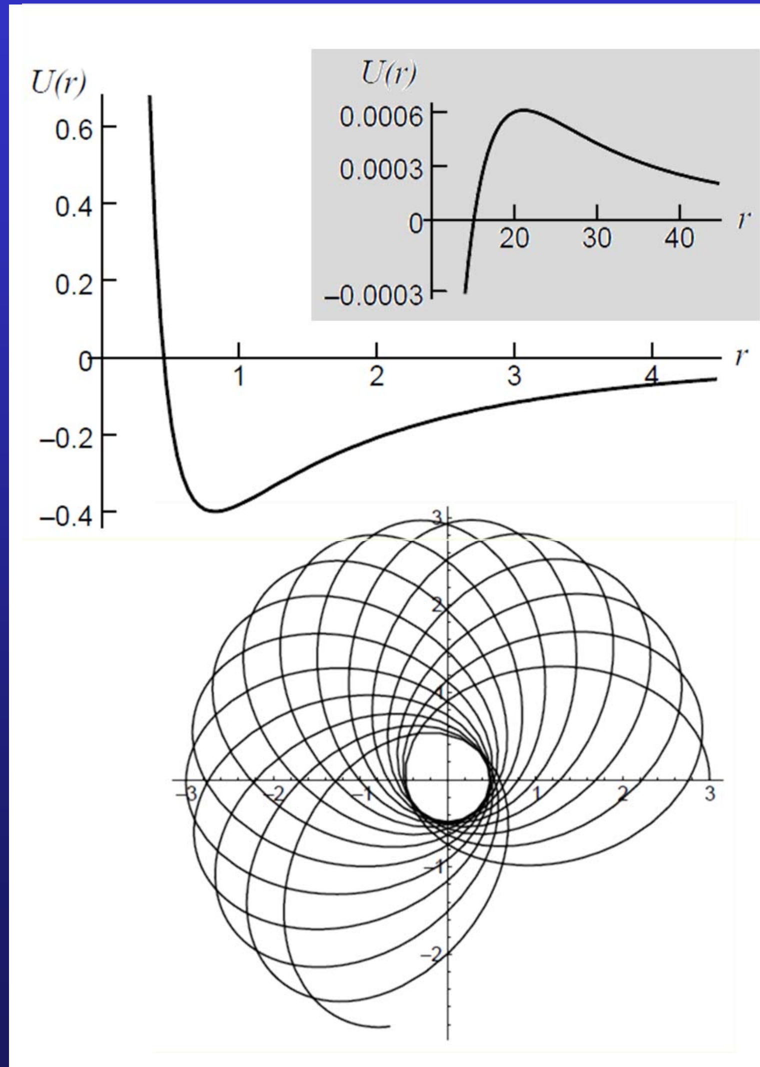
- angular momentum conserved

$$E = \frac{1}{2}m\dot{r}^2 + U_L(r)$$

where

$$U_L(r) = \frac{L^2}{2m}\frac{1}{r^2} - \frac{\alpha e^{-\kappa r}}{r}$$

YUKAWA POTENTIAL

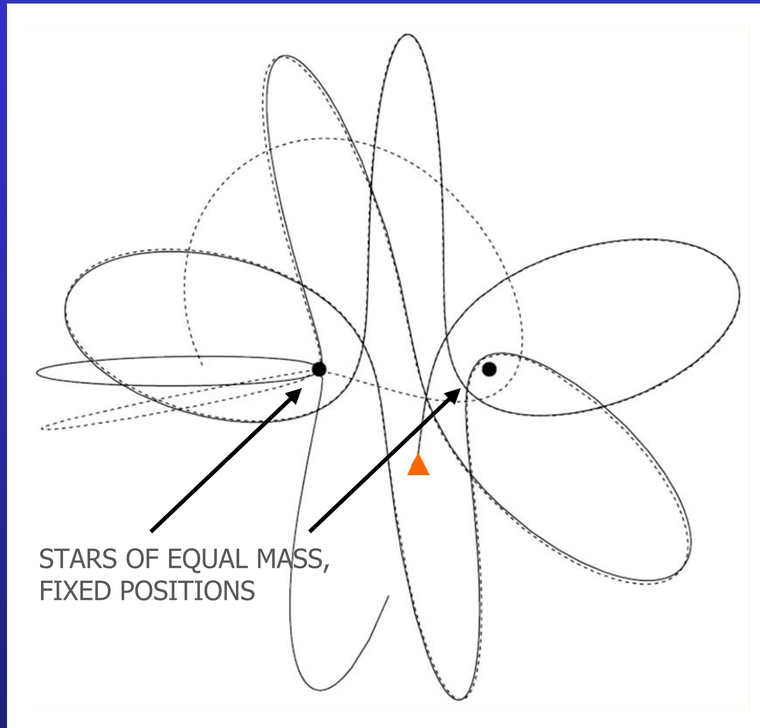


- attraction between nucleons
- precession of perihelion
- distinct allowed regions for small E
- alpha decay

Three-body problem

- notoriously intractable
- chaotic motion:
  - tiny difference in initial conditions results in very different trajectory

- total energy conserved
- angular momentum not conserved, since torque required to hold stars in place



Classical Mechanics

LINEAR MOTION OF SYSTEMS OF PARTICLES	centre of mass
	Newton's 2nd law for bodies (internal forces cancel)
	rocket motion
ANGULAR MOTION	rotations and infinitessimal rotations
	angular velocity vector, angular momentum, torque
	parallel and perpendicular axis theorems
	rigid body rotation, moment of inertia, precession
GRAVITATION & KEPLER'S LAWS	conservative forces, law of universal gravitation
	2-body problem, reduced mass
	planetary orbits, Kepler's laws
	energy, effective potential
NON-INERTIAL REFERENCE FRAMES	centrifugal and Coriolis terms
	Foucault's pendulum, weather patterns
NORMAL MODES	coupled oscillators, normal modes
	boundary conditions, Eigenfrequencies