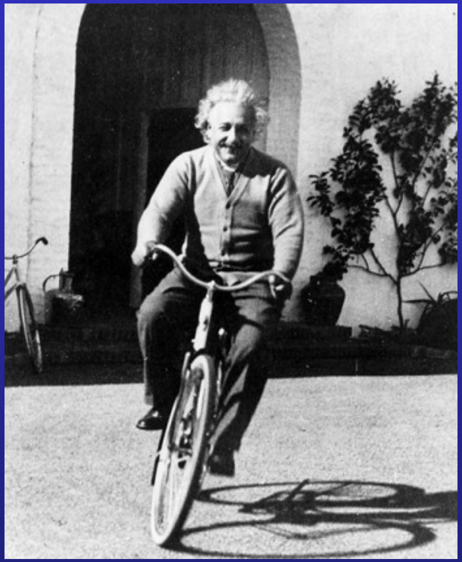
UNIVERSITY OF Southampton
School of Physics and Astronomy

Classical Mechanics

PHYS 2006

Tim Freegarde

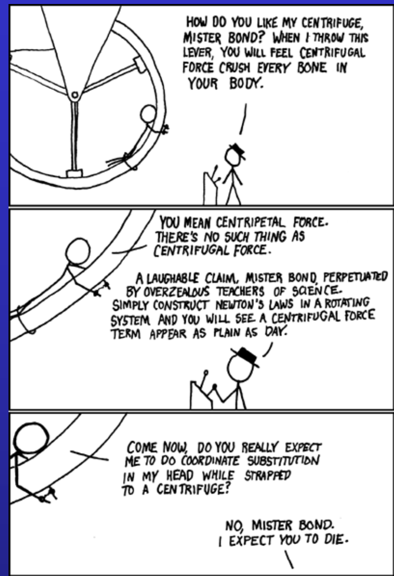


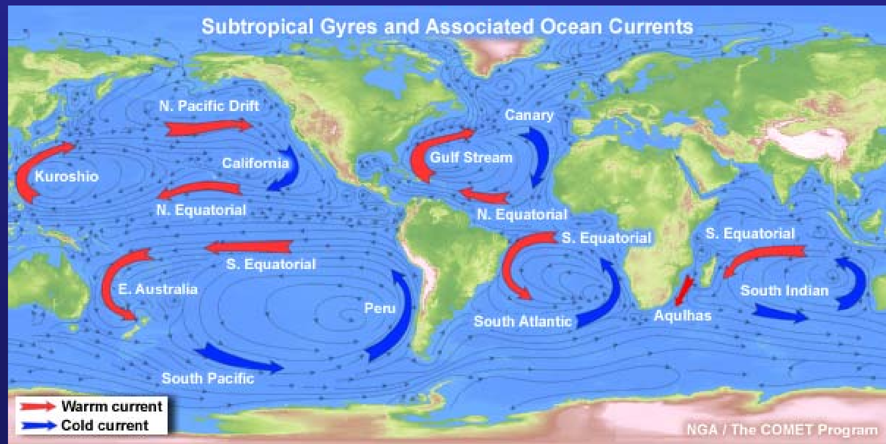
Classical Mechanics

LINEAR MOTION OF SYSTEMS OF PARTICLES	centre of mass
	Newton's 2nd law for bodies (internal forces cancel)
	rocket motion
ANGULAR MOTION	rotations and infinitessimal rotations
	angular velocity vector, angular momentum, torque
	parallel and perpendicular axis theorems
	rigid body rotation, moment of inertia, precession
GRAVITATION & KEPLER'S LAWS	conservative forces, law of universal gravitation
	2-body problem, reduced mass
	planetary orbits, Kepler's laws
	energy, effective potential
NON-INERTIAL REFERENCE FRAMES	centrifugal and Coriolis terms
	Foucault's pendulum, weather patterns
NORMAL MODES	coupled oscillators, normal modes
	boundary conditions, Eigenfrequencies

Rotating coordinate systems

- local coordinates not an inertial frame
- 'ficticious' forces:
  - centrifugal
  - Coriolis



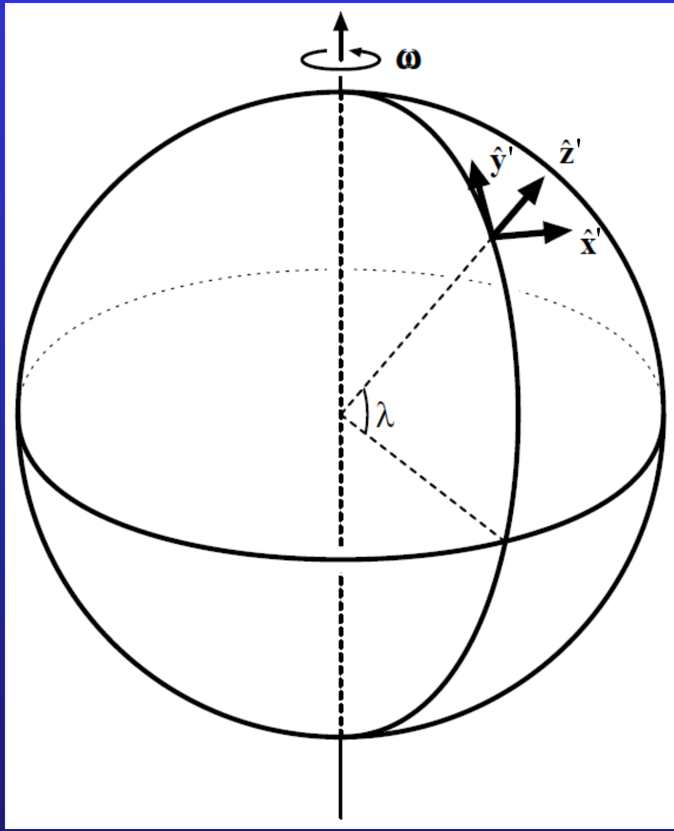


Rotating coordinate systems

$$\mathbf{a} = \underbrace{a_i\hat{\mathbf{i}} + a_j\hat{\mathbf{j}} + a_k\hat{\mathbf{k}}}_{\text{inertial frame}} = \underbrace{a_i'\hat{\mathbf{i}}' + a_j'\hat{\mathbf{j}}' + a_k'\hat{\mathbf{k}}'}_{\text{local, rotating frame}}$$

- differentiate twice, taking into account variation of local (rotating) unit vectors

$$m\ddot{\mathbf{r}} = \mathbf{F} \underbrace{- 2m\boldsymbol{\omega}\times\dot{\mathbf{r}}}_{\text{Coriolis}} \underbrace{- m\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})}_{\text{centrifugal}}$$



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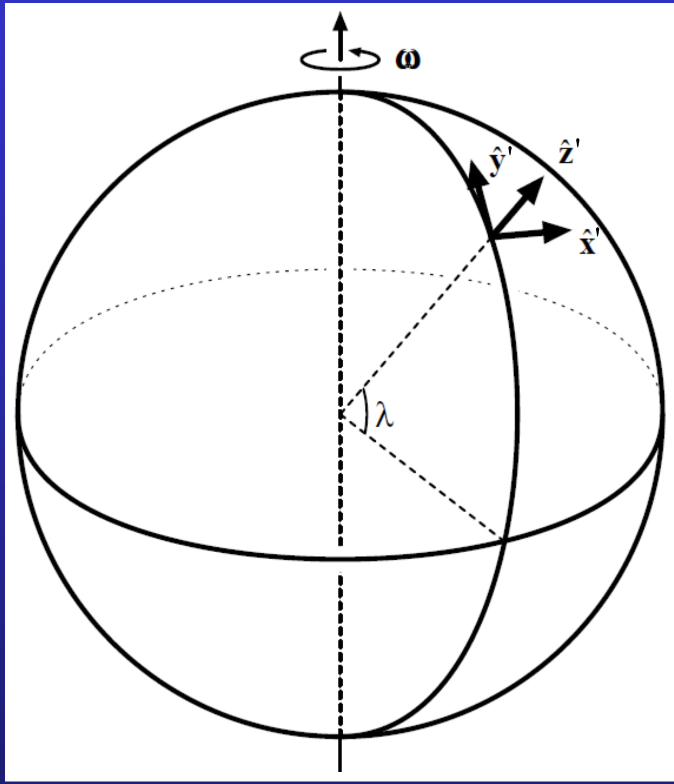

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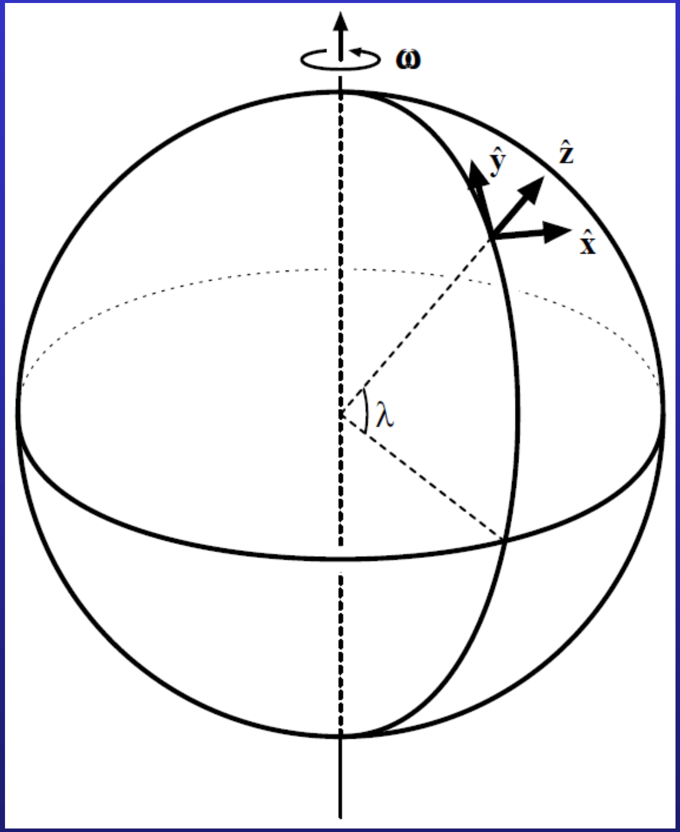
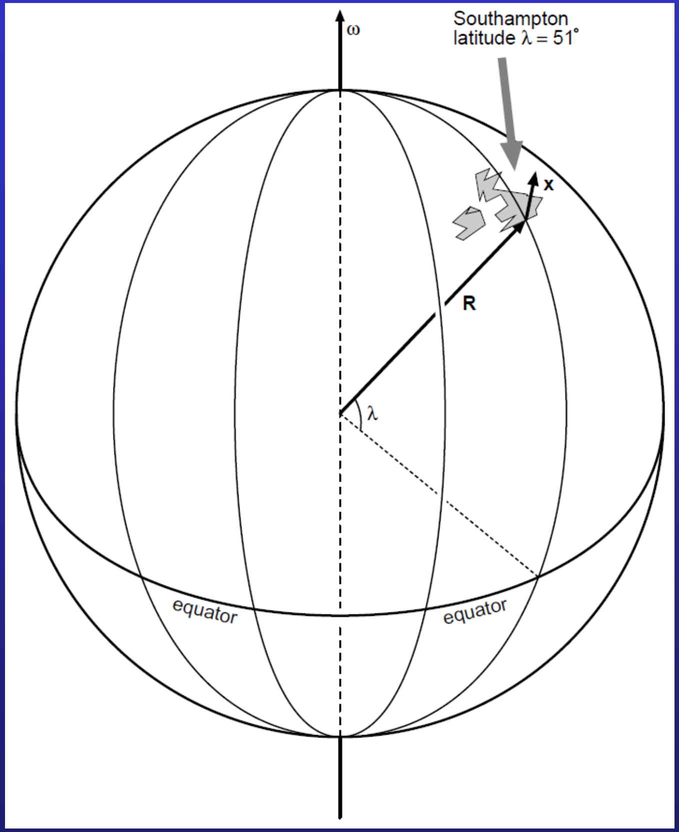
- in local coordinate frame $\{\hat{\mathbf{x}}', \hat{\mathbf{y}}', \hat{\mathbf{z}}'\}$, this becomes

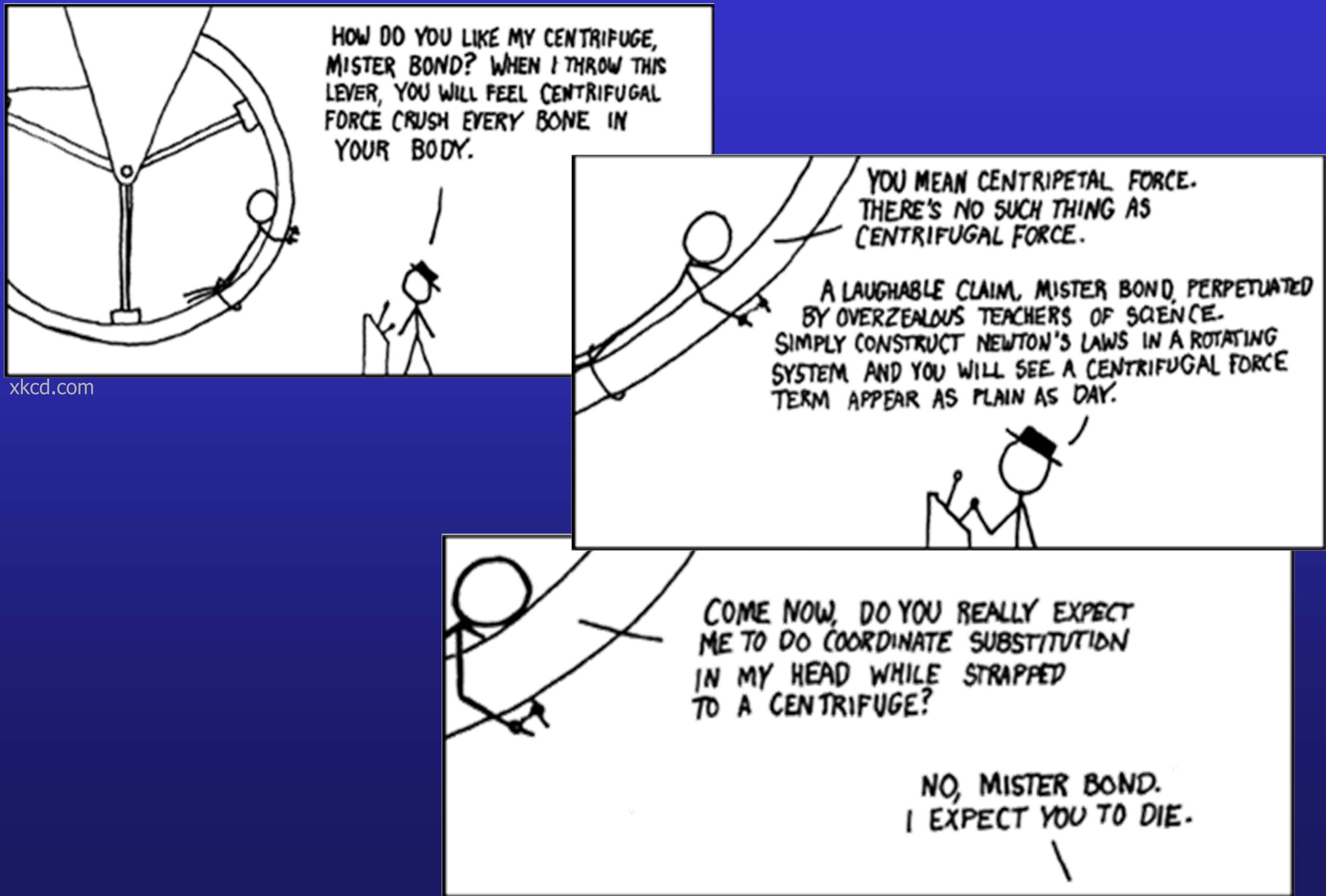
$$m\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix} = \mathbf{F}_{\text{ext}} + \underbrace{m\mathbf{g} + m\omega^2 R\cos\lambda\begin{pmatrix}0\\ -\sin\lambda\\ \cos\lambda\end{pmatrix}}_{m\mathbf{g}^*} - 2m\omega\begin{pmatrix}\dot{z}\cos\lambda - \dot{y}\sin\lambda\\ \dot{x}\sin\lambda\\ -\dot{x}\cos\lambda\end{pmatrix}$$

Rotating coordinate systems





Rotating coordinate systems



xkcd.com

Rotating coordinate systems

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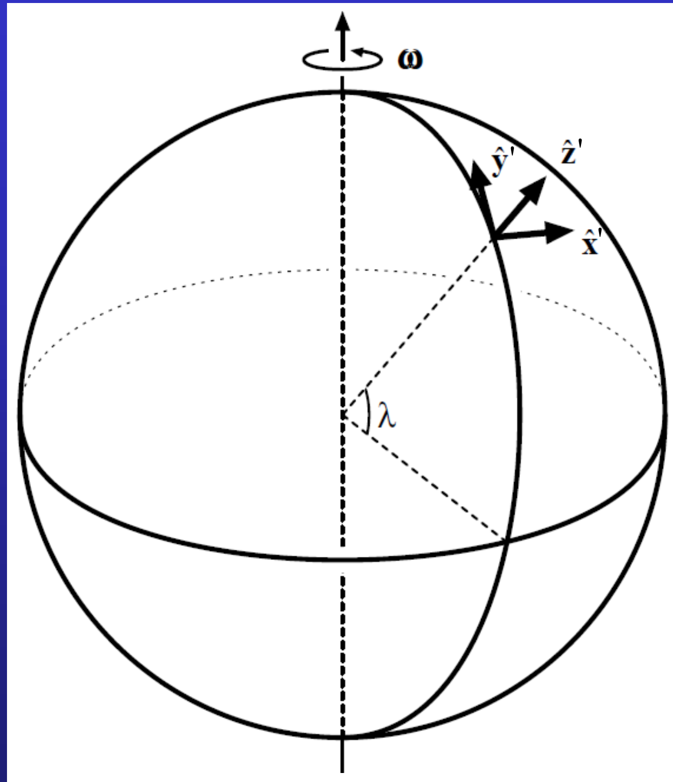

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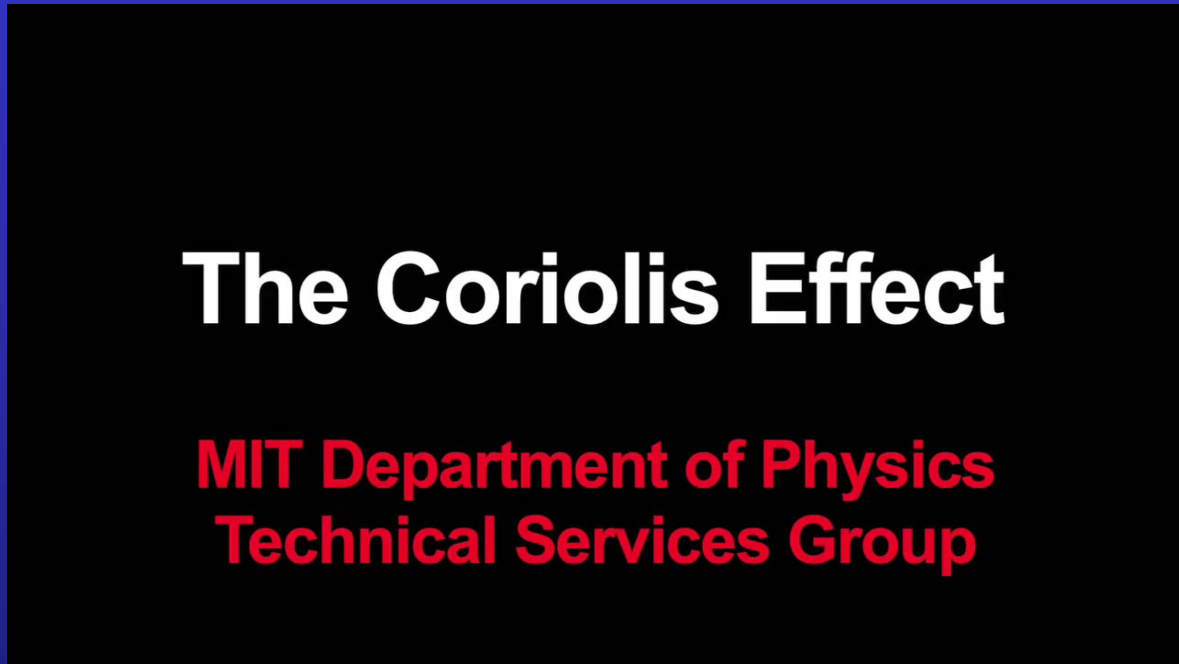
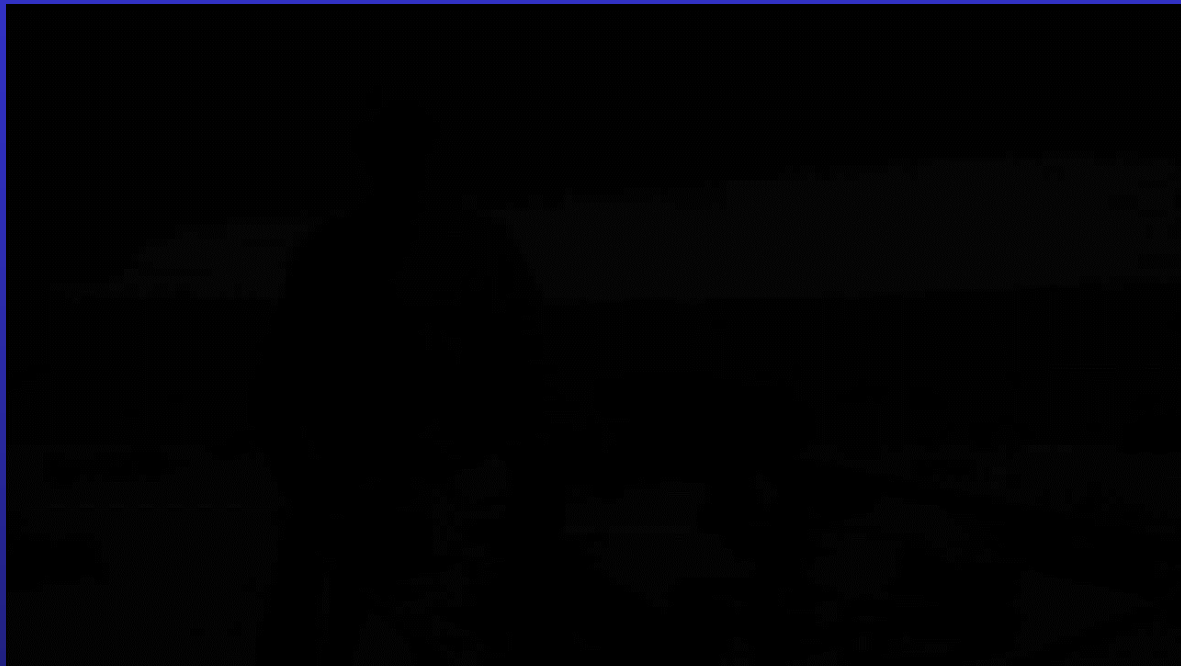
Coriolis effect

$$\mathbf{F}_{\text{Coriolis}} = -2m\,\boldsymbol{\omega} \times \dot{\mathbf{r}}$$



www.youtube.com/watch?v=dt_XJp77-mk

Coriolis effect



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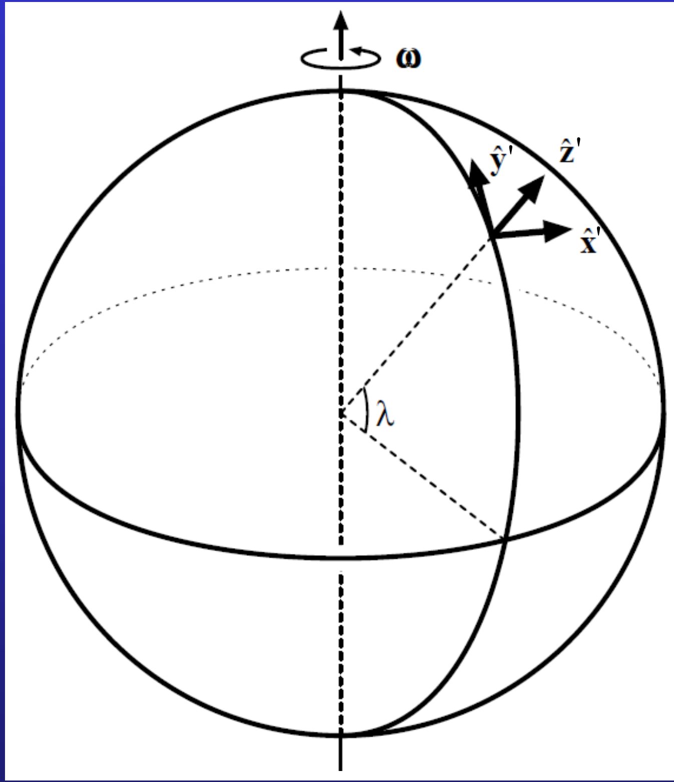

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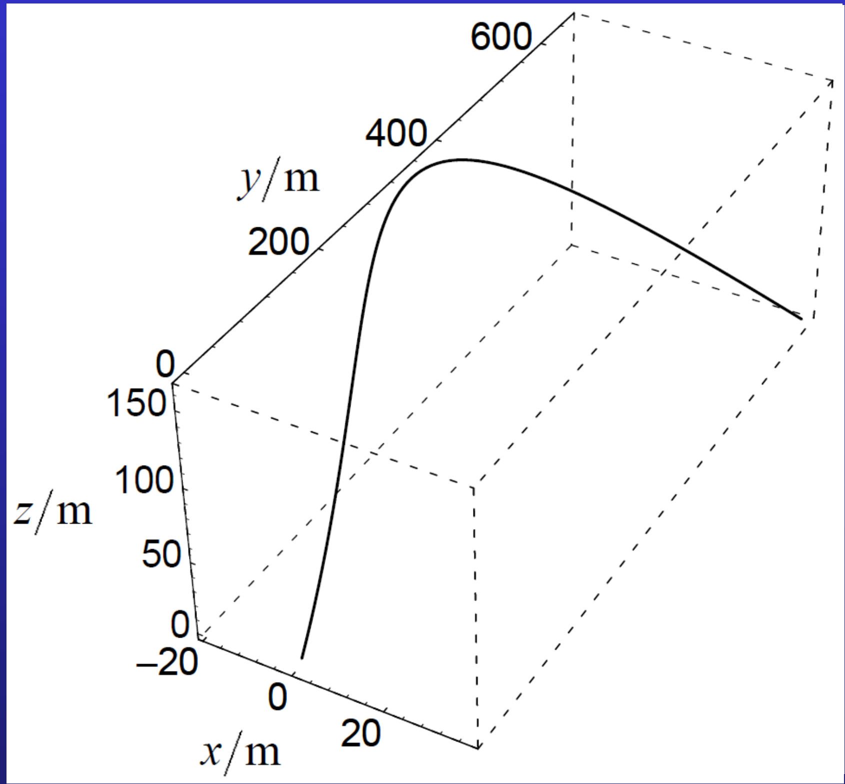
Cannon shell

- velocity varies during trajectory
- Coriolis correction hence changes direction along the shell's path



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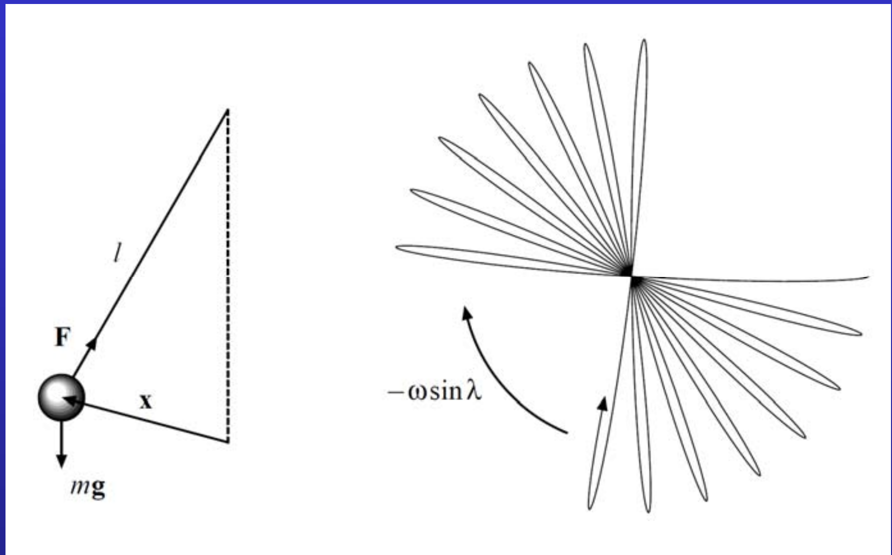
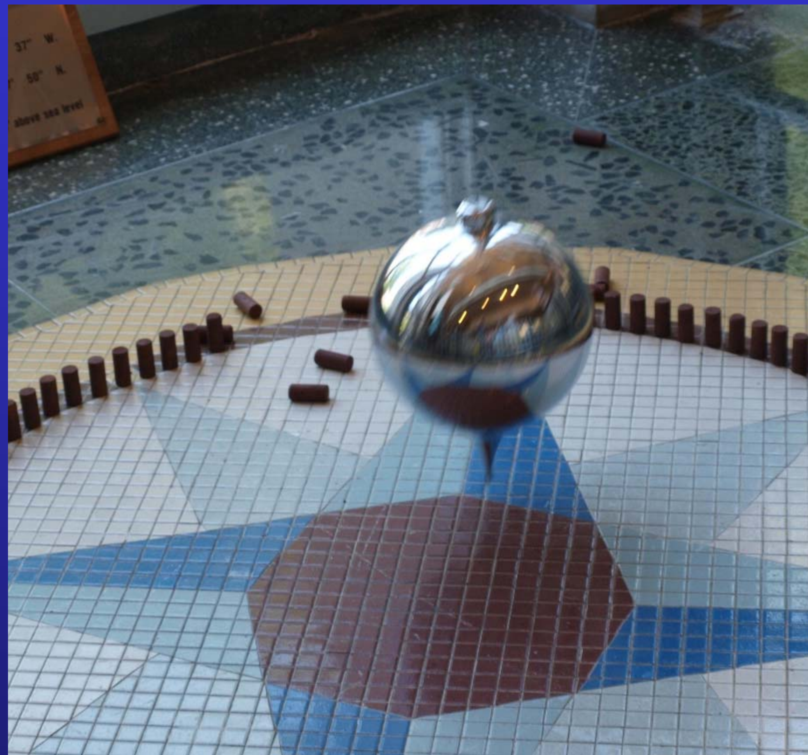
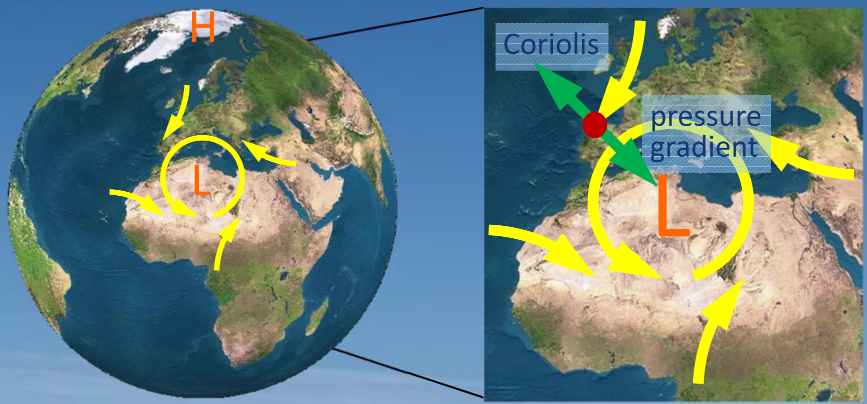
Foucault's pendulum

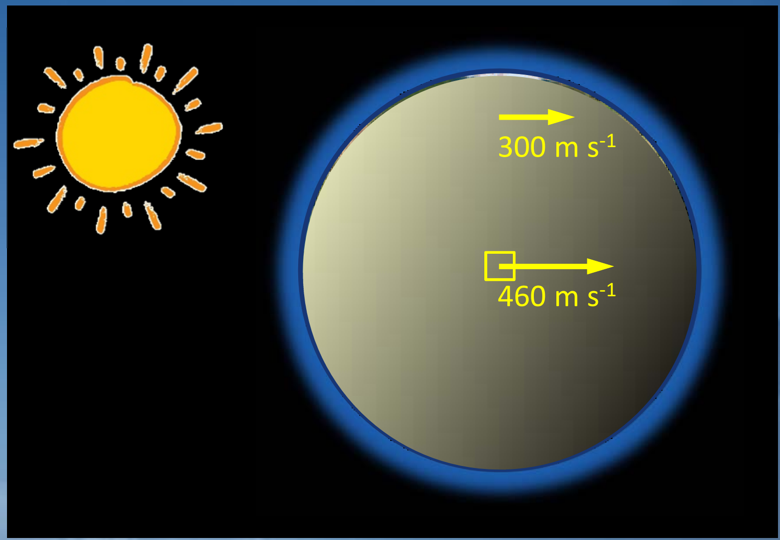




$$m\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} = -\omega_0^2\begin{pmatrix}x\\ y\end{pmatrix} - 2\omega\sin\lambda\begin{pmatrix}-\dot{y}\\ \dot{x}\end{pmatrix}$$

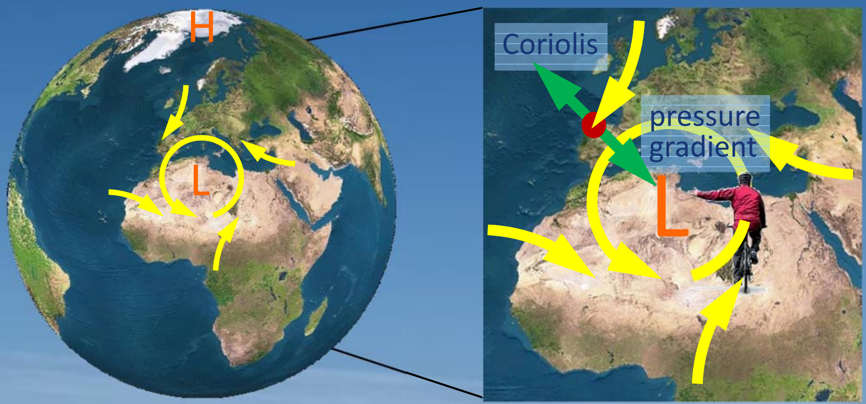
Coriolis effect

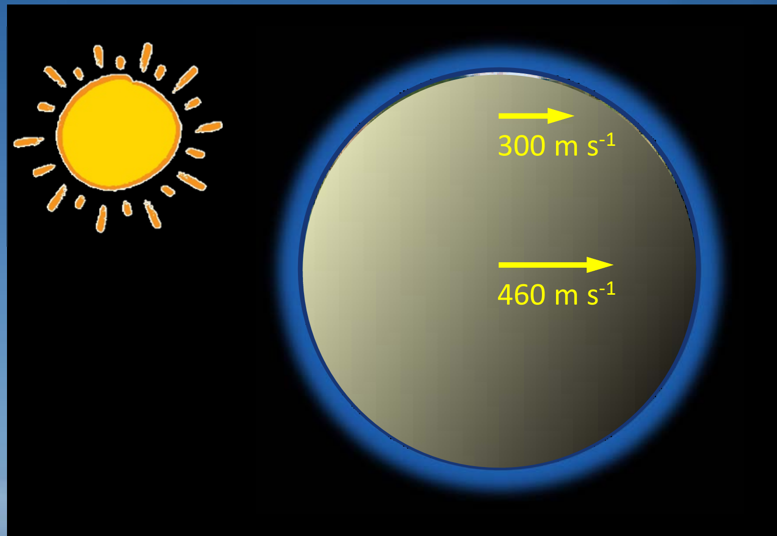




- tendency to turn right in N hemisphere
- to observer moving with Earth, appears as virtual force
- pressure gradient balances Coriolis & centrifugal forces
- at altitude, flow follows isobars around high/low pressure

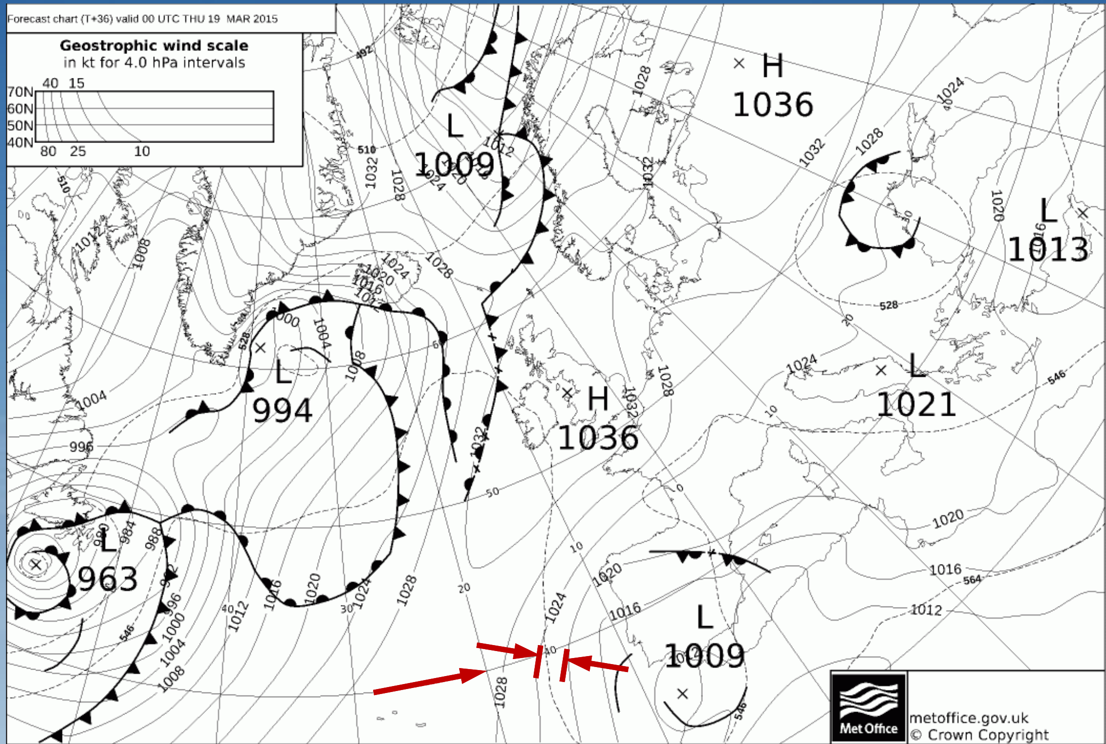
Buys-Ballot's law





- with the wind on your back,
  in the N hemisphere,
  the low pressure is on the left

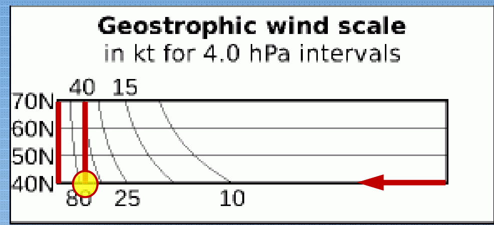
Geostrophic wind



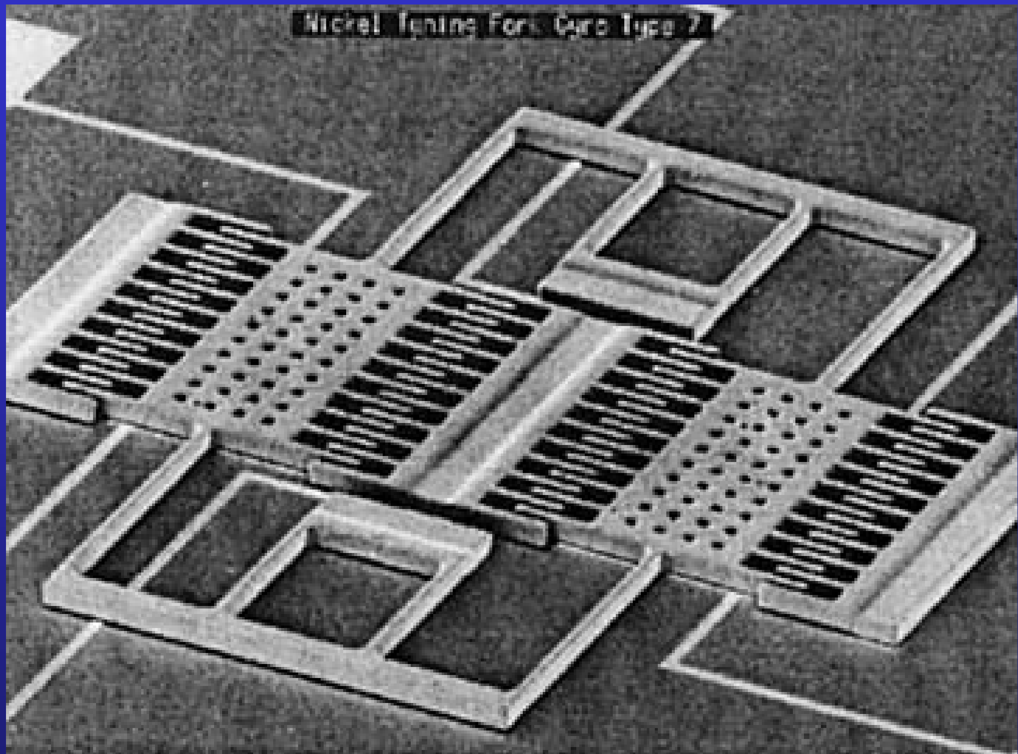
- isobars: pressure contours (4 hPa)
- cold/warm fronts divide air masses

GEOSTROPHIC WIND

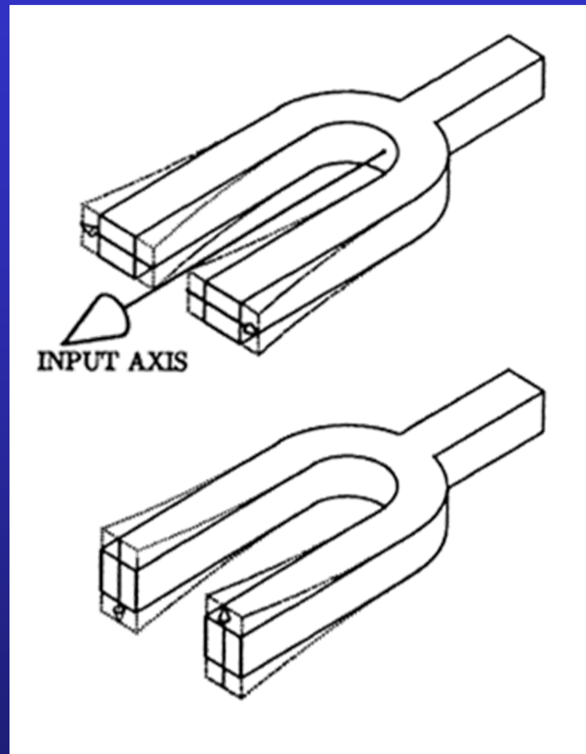
- closer isobars
  → higher Coriolis force
  → stronger wind



Vibrating structure gyroscope



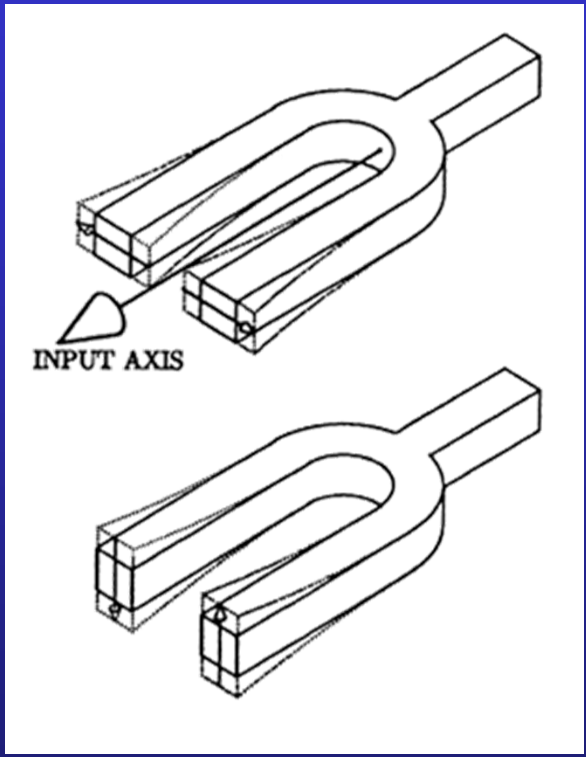
Weinberg, Bernstein, Cho, King, Kourepenis, Ward & Sohn,
Int Conf Gyrosc Technol & Nav (2), 9-87 (1995)



Grewal, Weill & Andres,
Global Positioning Systems, Inertial Navigation
and Integration (2007)

Vibrating structure gyroscope

Drosophila melanogaster, Shwetha Mureli



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and Integration (2007)

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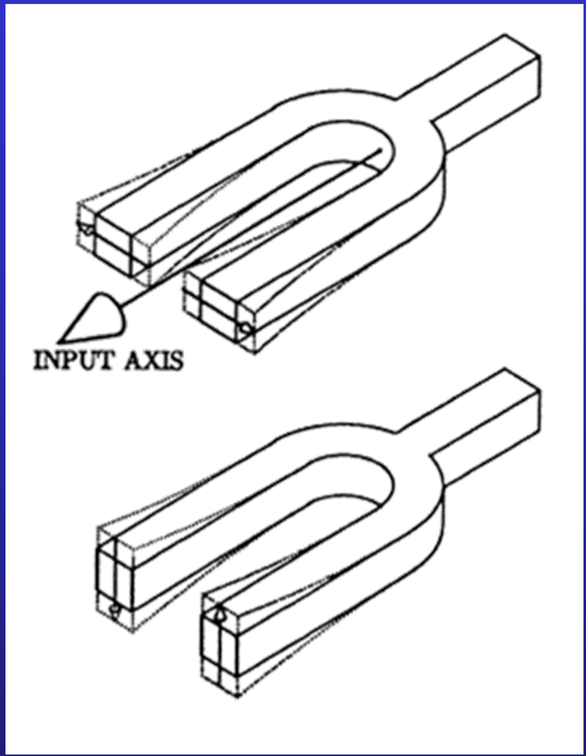
Life on Earth, BBC (1979)



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and Integration (2007)

- F W Meredith, Control of equilibrium in the flying insect, Nature **163**, 74 (1949)
- Michael Dickinson, How a fly flies, TED www.youtube.com/watch?v=e_44G-kE8lE

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