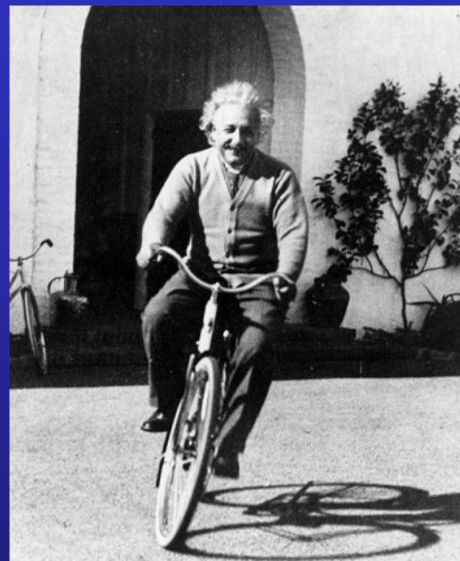
UNIVERSITY OF Southampton
School of Physics and Astronomy

Classical Mechanics

PHYS 2006

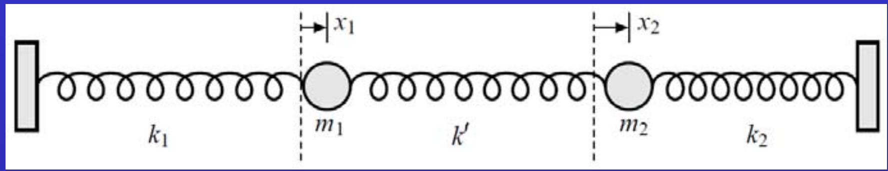
Tim Freegarde

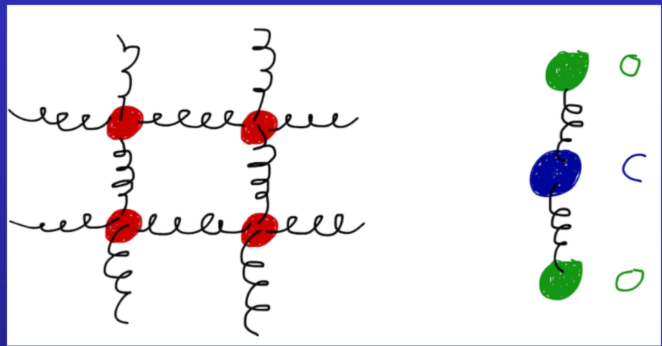


Classical Mechanics

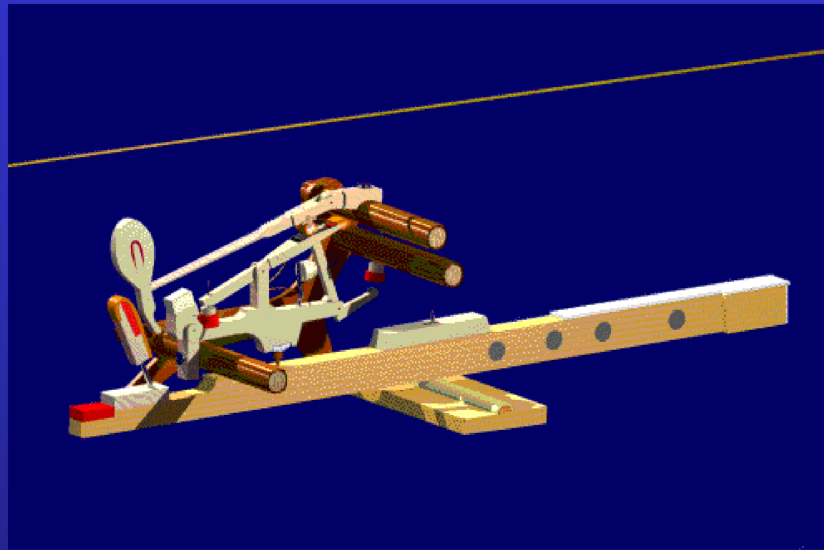
LINEAR MOTION OF SYSTEMS OF PARTICLES	centre of mass
	Newton's 2nd law for bodies (internal forces cancel)
	rocket motion
ANGULAR MOTION	rotations and infinitessimal rotations
	angular velocity vector, angular momentum, torque
	parallel and perpendicular axis theorems
	rigid body rotation, moment of inertia, precession
GRAVITATION & KEPLER'S LAWS	conservative forces, law of universal gravitation
	2-body problem, reduced mass
	planetary orbits, Kepler's laws
	energy, effective potential
NON-INERTIAL REFERENCE FRAMES	centrifugal and Coriolis terms
	Foucault's pendulum, weather patterns
NORMAL MODES	coupled oscillators, normal modes
	boundary conditions, Eigenfrequencies

Coupled oscillators





Ethan Neil, U Colorado



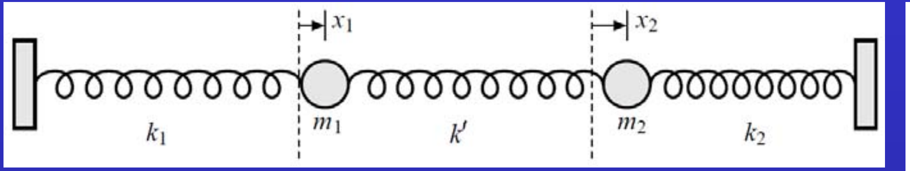
K Wayne Land www.musicresourcesusa.com

- normal modes

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} \exp i\omega t$$

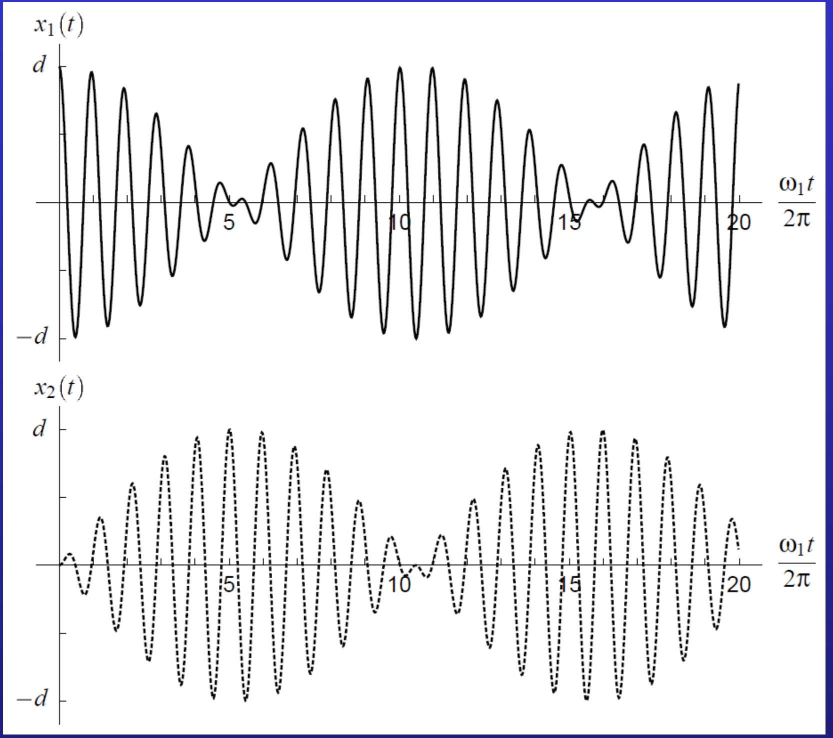
- eigenmodes, eigenfrequencies

Coupled oscillators



SYMMETRIC

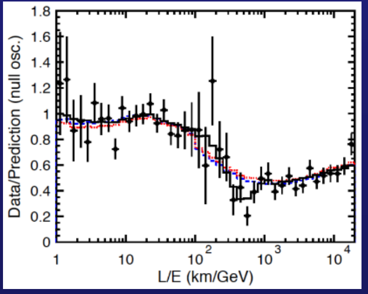
ANTI-SYMMETRIC

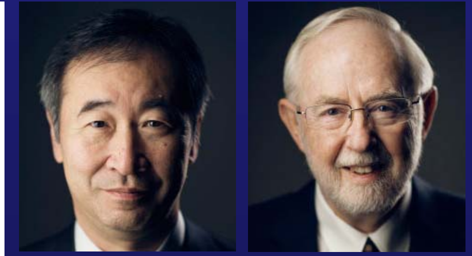


- beating when normal modes superposed

- neutrino oscillations between flavours

- avoided crossings, ac Stark effect, …





T Kajita, A McDonald (Nobel 2015)

Coupled oscillators

www.youtube.com/watch?v=CjJVBvDNxcE

Triatomic molecule

- determine force on, hence acceleration of, each atom

$$\begin{pmatrix} m_1\ddot{x}_1 \\ m_2\ddot{x}_2 \\ m_3\ddot{x}_3 \end{pmatrix} = \begin{pmatrix} -k_1 & k_1 & 0 \\ k_1 & -(k_1+k_3) & k_3 \\ 0 & k_3 & -k_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

- seek harmonic solutions to

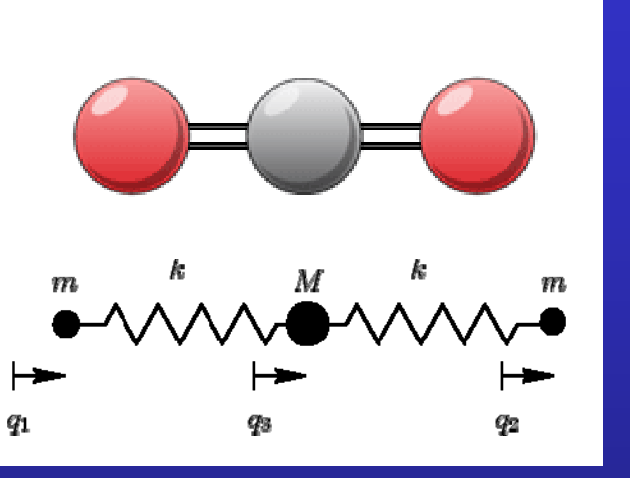
$$\begin{pmatrix} m_1\ddot{z}_1 \\ m_2\ddot{z}_2 \\ m_3\ddot{z}_3 \end{pmatrix} = \begin{pmatrix} -k_1 & k_1 & 0 \\ k_1 & -(k_1+k_3) & k_3 \\ 0 & k_3 & -k_3 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

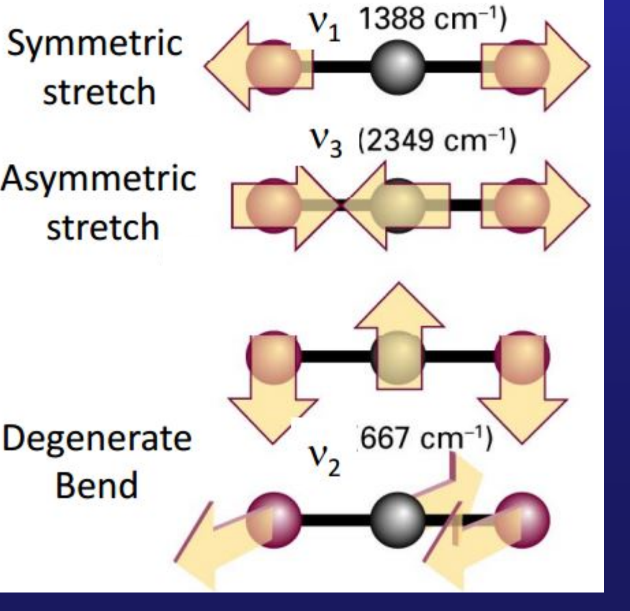
where $x_n = \mathrm{Re}(z_n)$

- set $m_1 = m_3 = m$, $m_2 = M$ and $k_1 = k_3 = k$

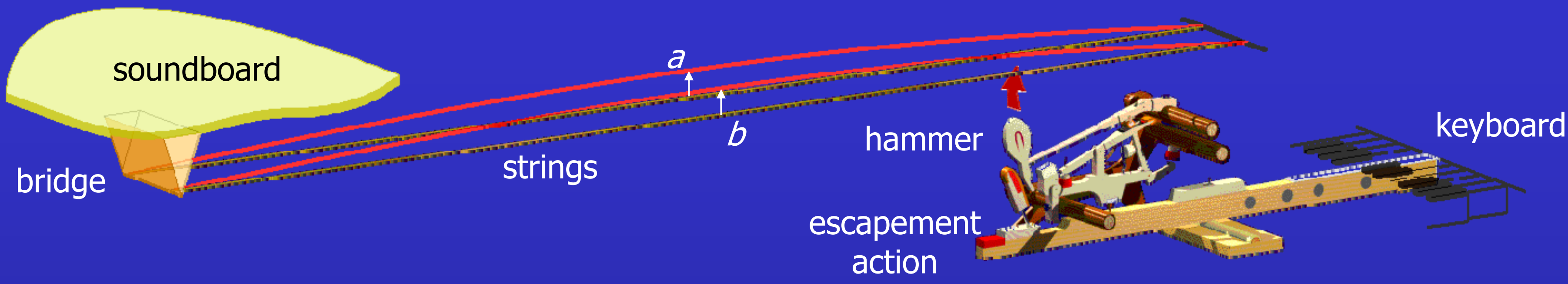
- solutions are roots of

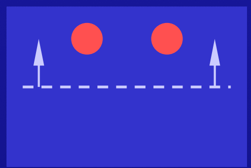
$$\underbrace{\omega^2}_{\text{TRANSLATION}}\ \underbrace{(m\omega^2 - k)}_{\text{SYMMETRIC}}\ \underbrace{(m^2\omega^2 - k(2m+M))}_{\text{ASYMMETRIC}} = 0$$





Coherent control: the pianoforte





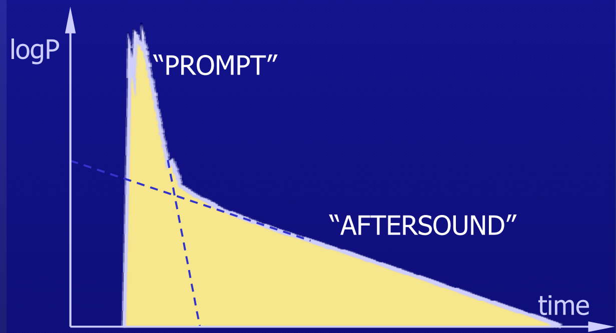
ψ_s

- strong coupling
  $\Rightarrow$ rapid decay

- strike strings together to produce almost pure ψ_s

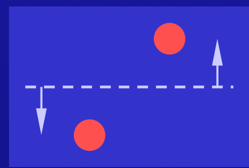
FORTE

MODES OF
COUPLED MOTION



- strike one string - *una corda* - to produce equal superposition
- reinforced "aftersound" is audible even when played softly

PIANO



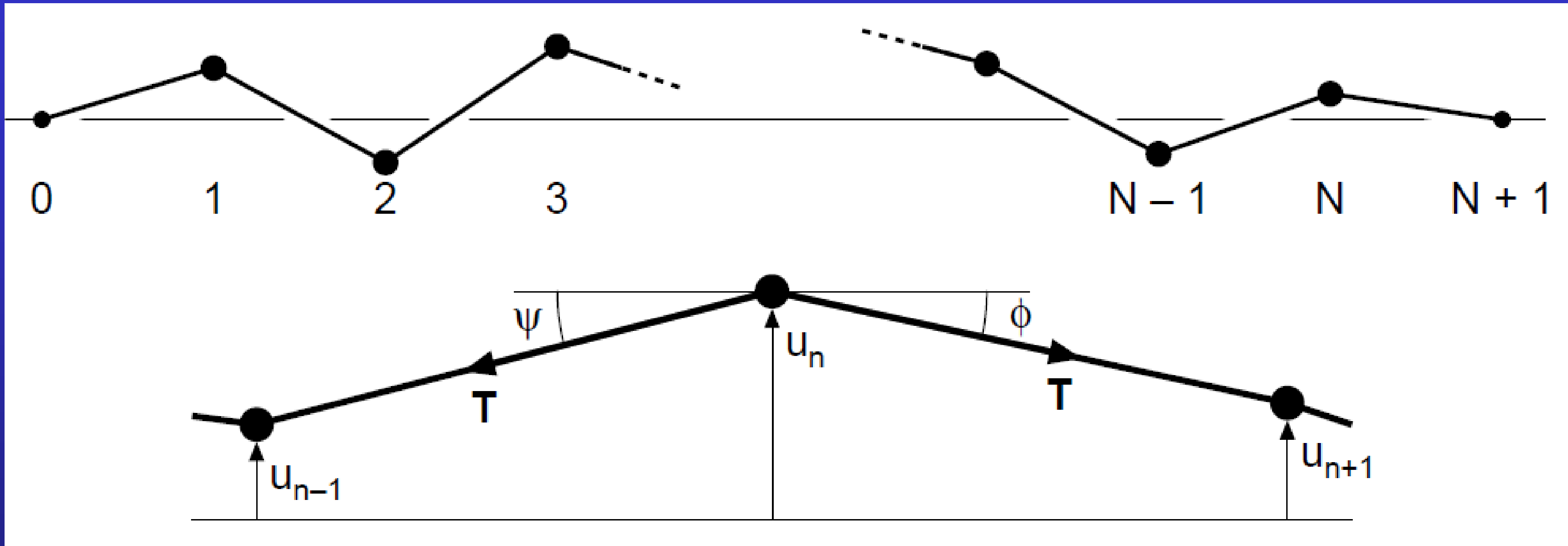
ψ_a

- weak coupling
  $\Rightarrow$ slow decay

- strike strings together from opposite sides to produce almost pure ψ_a

G Weinreich, Sci Am (Jan 1979) pp94-102

Normal modes of a beaded string



Normal modes of a beaded string

- determine force on, hence acceleration of, each bead

$$m_n \ddot{u}_n = -T(\sin\psi + \sin\phi)$$
$$\approx \frac{T}{a}\left[u_{n+1} - 2u_n + u_{n-1}\right]$$

- seek harmonic solutions to

$$\ddot{z}_n = \frac{T}{m_n a}\left[z_{n+1} - 2z_n + z_{n-1}\right]$$

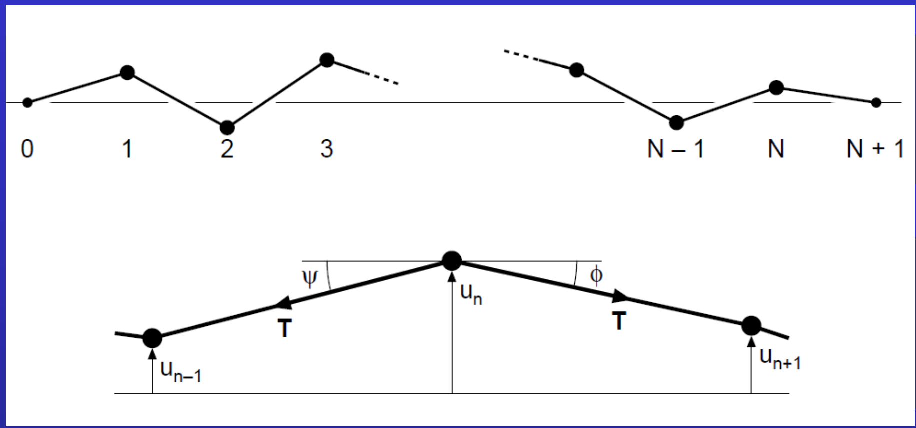
where $u_n = \mathrm{Re}(z_n)$

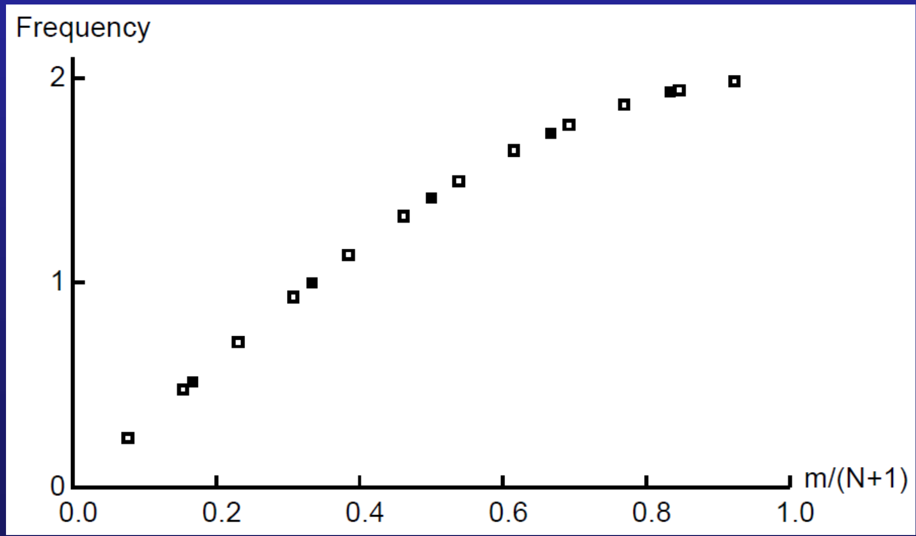
- assume translational symmetry

$$u_n = h u_{n-1}; m_n = m$$

- dispersion relation

$$\omega = \pm\sqrt{\frac{4T}{ma}}\sin\left(\frac{ka}{2}\right)$$





Normal modes of a finite beaded string

- determine force on, hence acceleration of, each bead

$$m_n \ddot{u}_n = -T(\sin\psi + \sin\phi) \approx \frac{T}{a}\left[u_{n+1} - 2u_n + u_{n-1}\right]$$

- seek harmonic solutions to

$$\ddot{z}_n = \frac{T}{m_n a}\left[z_{n+1} - 2z_n + z_{n-1}\right]$$

where $u_n = \mathrm{Re}(z_n)$

- assume translational symmetry

$$u_n = h u_{n-1};\, m_n = m$$

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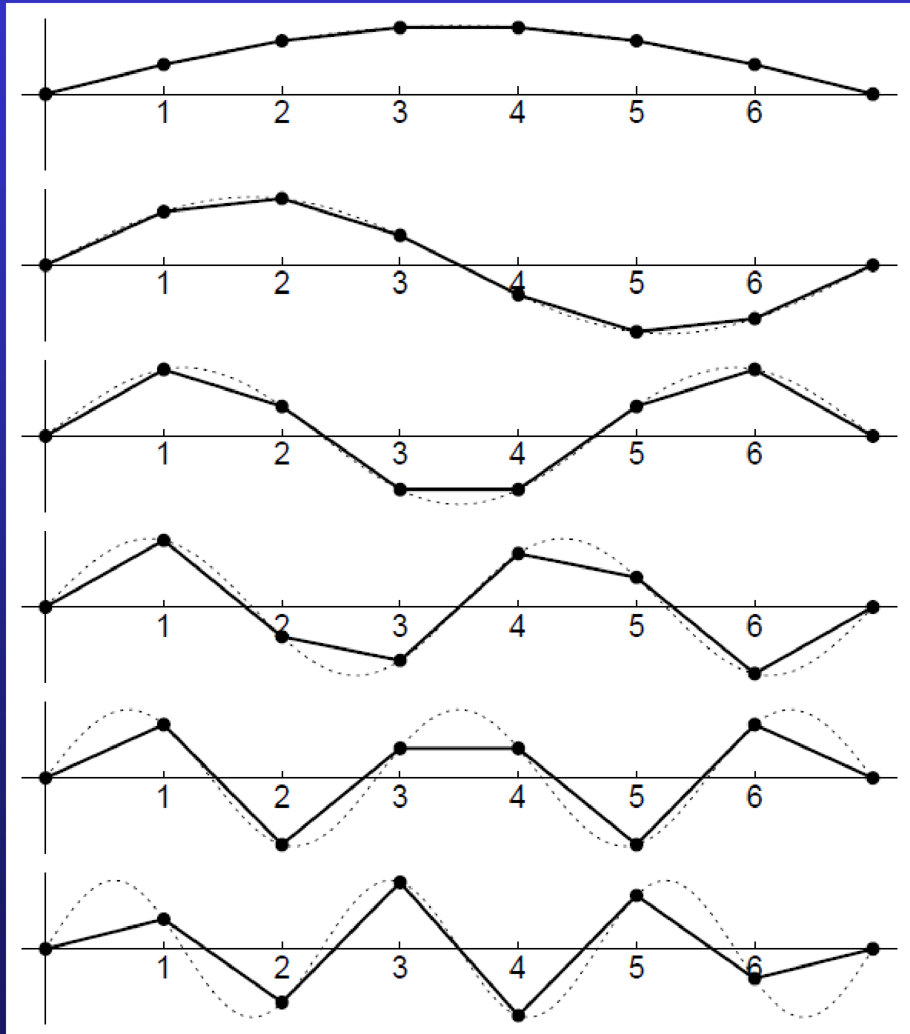
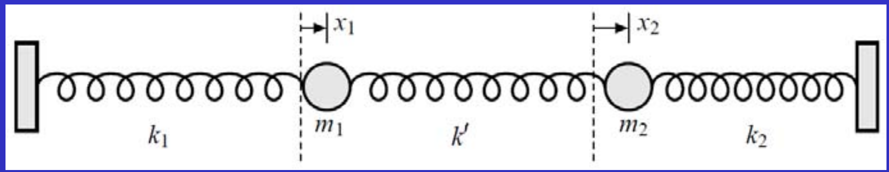


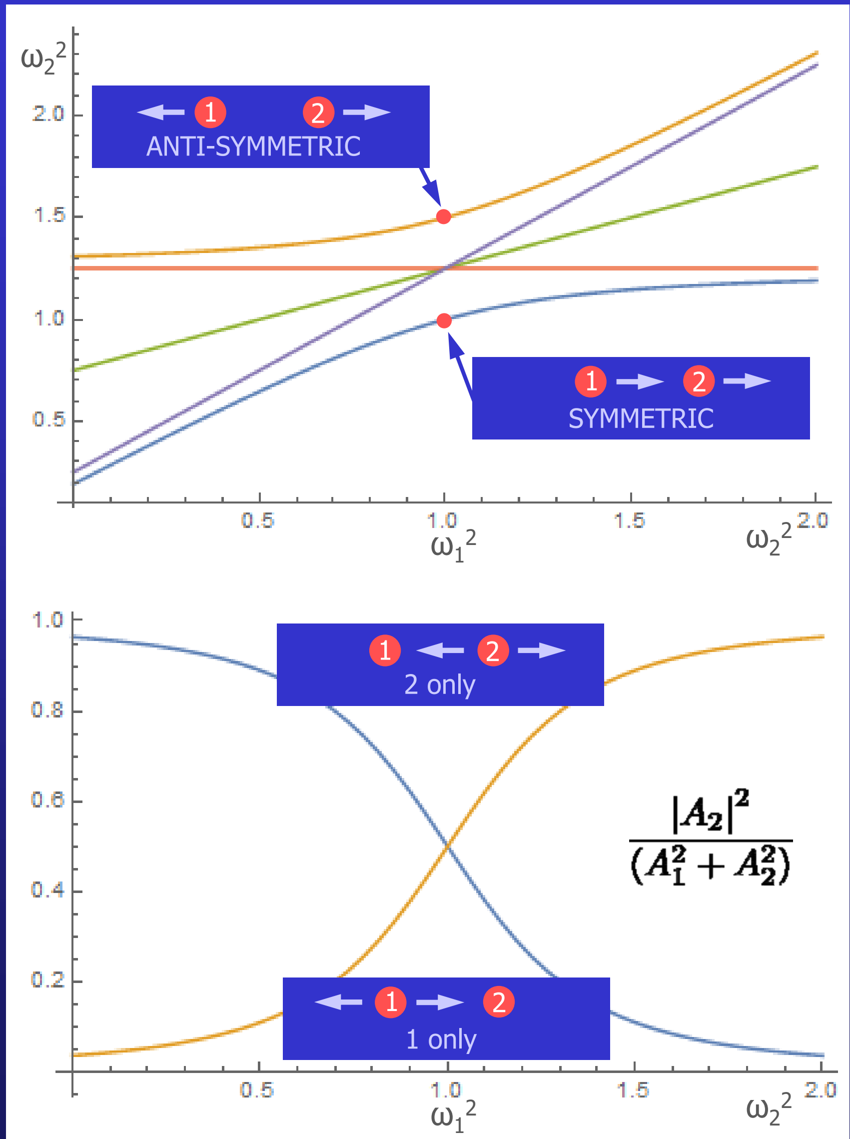
Avoided crossings



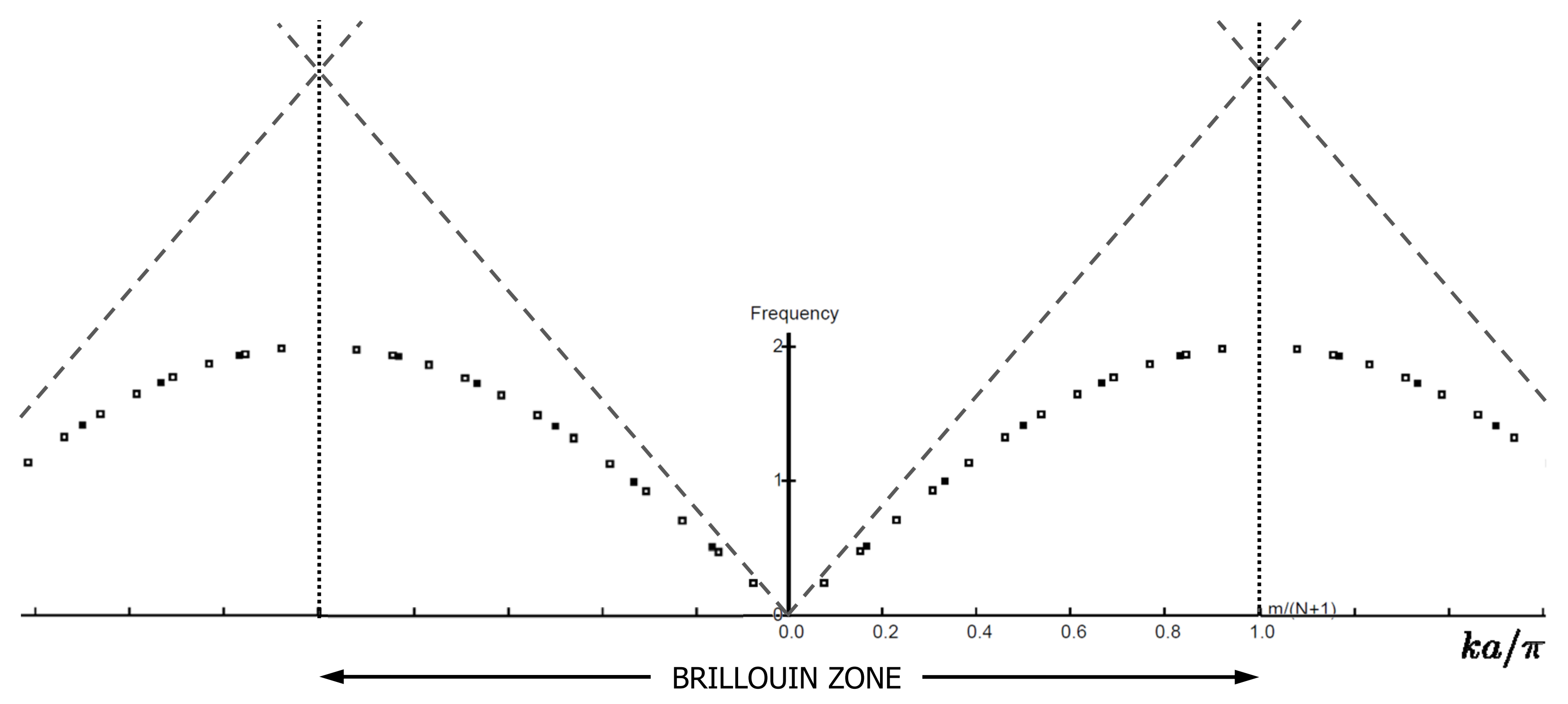
- if $k'=0$, oscillator frequencies $\omega_{1,2}^2 = \dfrac{k_{1,2}}{m_{1,2}}$
- fix ω_1 and vary ω_2 …somehow…
- when $\omega_1 \ll \omega_2$ or $\omega_1 \gg \omega_2$, all mode energy all in one oscillator
- when $\omega_1 \approx \omega_2$, antisymmetric or symmetric mode: energy divided equally
- frequency splitting depends upon coupling strength k'

ADIABATIC PASSAGE

- sweep ω_2 from $\ll \omega_1$ to $\gg \omega_1$ (or reverse)
- complete energy transfer from one oscillator to the other



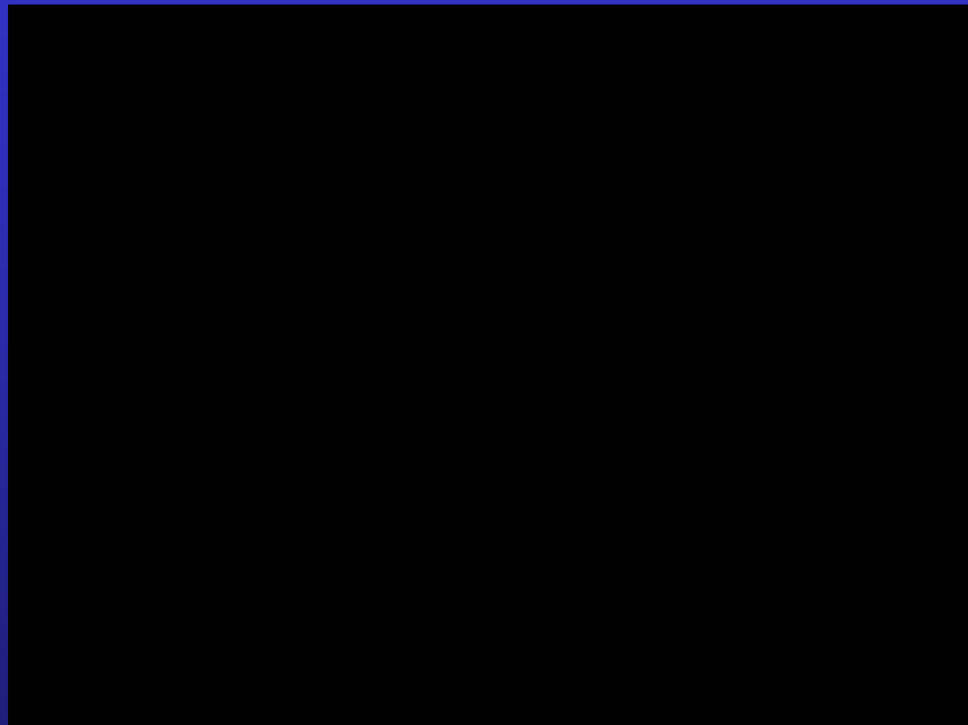
Avoided crossings



NYQUIST SAMPLING THEOREM

- wavenumbers beyond Brillouin zone repeat motion of lower values
- forward and backward travelling waves act as coupled oscillators
- at $k = \dfrac{\pi}{a}$, forward and backward travelling waves degenerate → avoided crossing

Coupled oscillators



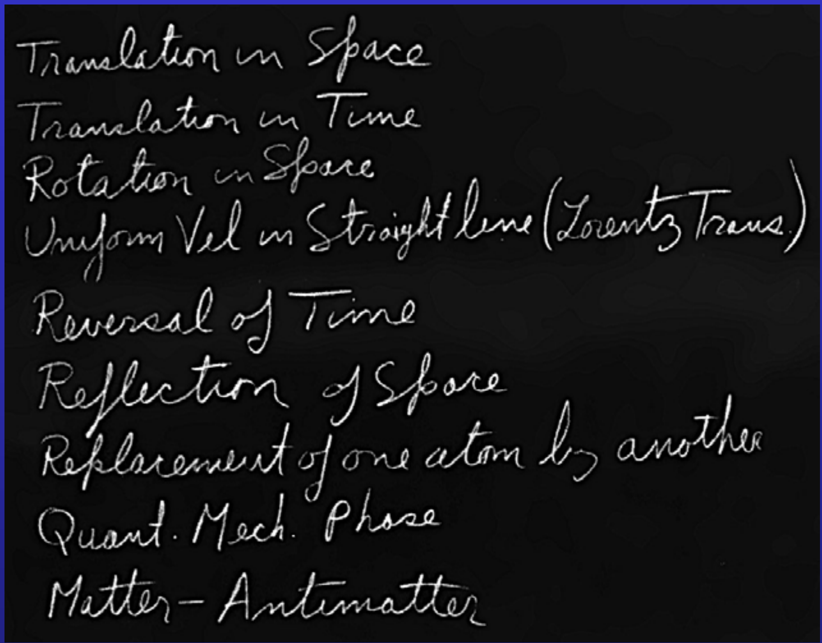
www.youtube.com/watch?v=V87VXA6gPuE

Symmetries and conserved quantities

symmetry		quantity / label
translation in space	$\mathbf{r} \to \mathbf{r}+\delta\mathbf{r}$	momentum
translation in time	$t \to t+\delta t$	energy
rotation	$\vartheta \to \vartheta+\delta\vartheta$	angular momentum
change of inertial frame	$\mathbf{v} \to \mathbf{v}+\delta\mathbf{v}$	centre of mass
reversal of time	$t \to -t$	entropy; 'T'
reflection in space	$x \to -x,\ y \to -y, z \to -z$	parity; 'P'
matter-antimatter interchange	$p \to \overline{p}$	'charge conj.'; 'C'
change of quantum mechanical phase	$\psi \to \mathrm{e}^{\mathrm{i}\varphi}\psi$	electrical charge
exchange of identical particles	$\{1,2\} \to \{2,1\}$	'exchange'
spatial periodicity	$x \to x+na$	quasi-momentum

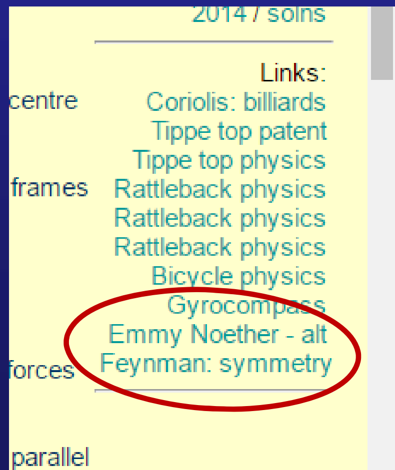
Emmy Noether (1882-1935)

Coupled oscillators



- Feynman Lectures in Physics I, chapter 52

Richard P Feynman (1918-1988)



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