Classical Mechanics - exercise sheet 5

This week's exercises continue to examine rotational motion and begin the topic of gravitation. Please submit your solutions through the Blackboard site by 3pm on **Friday 13th November**.

Reading

Continue your reading about gyroscopic motion and precession in your favourite textbook. e.g.

Fowles & Cassiday	*Analytical Mechanics* (7th ed.)	sections 9.4-9.10
Chow	*Classical Mechanics* (2nd ed.)	sections 12.4, 12.7-12.8
French & Ebison	*Introduction to Classical Mechanics*	pp. 255-295
Kibble & Berkshire	*Classical Mechanics* (5th ed.)	sections 9.3-9.10
Thornton & Marion	*Classical Dynamics* (5th ed.)	chapter 11

1 Disc inertia **(5 marks)**

Show that the moment of inertia of a thin circular disc of mass m and radius a is

a) (1/4) ma^2 for rotation about its diameter

b) (1/2) ma^2 for rotation about its axis of rotational symmetry.

2 Torque **(5 marks)**

A particle of mass m follows a path given by

$$\mathbf{r} = \left(x_0 + at^2\right)\hat{\mathbf{i}} + bt\hat{\mathbf{j}} + ct^3\hat{\mathbf{k}}$$

where x_0, a, b and c are constants and t is the time.

a) Find the angular momentum $\mathbf{L}$ of the particle about the origin.

b) Find the force $\mathbf{F}$ required to produce this motion and verify explicitly that $d\mathbf{L}/d\mathbf{t} = \mathbf{r} \times \mathbf{F}$.

3 Looping the loop **(5 marks)**

A glider performs a 'loop', beginning at the bottom where it moves horizontally with speed v_1, and following a perfect circle of radius a in the vertical plane. Its speed at the top of the loop is v_2, and it may be assumed that no energy is lost in the manoeuvre. If the glider is 'weightless' at the top of the loop, show that at the bottom of the loop the acceleration is $5g$ and the wings must generate a lift force equal to six times the glider's normal weight.

4 Gravity and prospecting for minerals **(5 marks)**

a) Show that the gravitational field due to a horizontal uniform thin disc of thickness d, radius R and density ρ, at a distance h vertically above the centre of the disc, has magnitude

$$2\pi G\rho d\left(1 - \frac{h}{\left(R^2 + h^2\right)^{1/2}}\right).$$

b) A pendulum clock in the centre of a large room is observed to keep correct time. How many seconds per year will the clock gain if the floor beneath is flooded to a depth of 15 cm with sea water of density 1 025 kg m^{-3}?

c) The clock is then moved to a position directly above a wide seam of iron ore that is 120 m thick and 500 m below the Earth's surface. Whereas the typical density of the Earth's crust is around 2 700 kg m^{-3}, iron ore has a density of about 3 000 kg m^{-3}. How many seconds per year will the clock now gain or lose?

[Newton's gravitational constant is $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.]