Classical Mechanics - exercise sheet 6

These exercises revise the first part of the Classical Mechanics course. Please submit your solutions through the Blackboard site by 3pm on **Friday 20th November**.

Reading

Revise the relevant sections of your favourite textbook. e.g.

Fowles & Cassiday	*Analytical Mechanics* (7th ed.)	chapters 1, 2, 6-9
Chow	*Classical Mechanics* (2nd ed.)	chapters 1, 2, 3, 6, 12
French & Ebison	*Introduction to Classical Mechanics*	chapters 1-5, 9, 10
Kibble & Berkshire	*Classical Mechanics* (5th ed.)	chapters 1, 4, 7-9
Thornton & Marion	*Classical Dynamics* (5th ed.)	chapters1, 8, 9, 11

1 Rotational motion (5 marks)

a) Define *torque* and state how it affects the angular momentum if the vectors are (i) parallel, (ii) perpendicular.

b) Define the *moment of inertia* about a fixed rotation axis, and its effect upon the response to a torque.

2 Freewheeling (5 marks)

Suppose that you are freewheeling down a hill on a bicycle.

a) Ignoring air resistance and friction within the bicycle mechanisms, show that your acceleration will be

$$\frac{Mg\sin\alpha}{M+2I/a^2}$$

where M is the total mass of the bicycle and rider, g the acceleration due to gravity, α the angle between the slope of the hill and the horizontal, I is the moment of inertia of each bicycle wheel, and a is the wheel radius.

b) Suppose now that you and a friend are freewheeling down the hill on identical bicycles. If all things are equal except that your friend is heavier than you, explain whether you both accelerate at the same rate, or whether one of you overtakes the other.

c) Your friend now swaps his/her bicycle for one that has larger wheels but which is otherwise the same in all other respects, including the wheel mass. What difference does this make to your friend's acceleration?

3 Laplace-Runge-Lenz vector (5 marks)

For motion under a central conservative force, the total energy and the angular momentum **L** are both conserved. If the force obeys an inverse-square law force with potential energy V(**r**) = -*k*/*r*, the *Laplace-Runge-Lenz* vector,

$$\mathbf{A}=\mathbf{p}\times\mathbf{L}-mk\hat{\mathbf{r}},$$

is also conserved. Show that **A** is perpendicular to **L** and, by recalling the standard orbit equation

$$\frac{l}{r}=1+e\cos\vartheta,$$

where $l = L^2/mk$, express the eccentricity e in terms of the length A of the Laplace-Runge-Lenz vector.

4 Gravity underground (5 marks)

The magnitude g of the acceleration due to gravity is found to be greater down a mineshaft than it is on the Earth's surface. Show that this can be explained if

$$\rho_s<\frac{2}{3}\rho_{av}$$

where ρ_s is the density of the Earth at its surface and ρ_{av} is the Earth's average density.

[Assume the Earth be spherically symmetric, and neglect any effects due to its rotation.]