Classical Mechanics - exercise sheet 7

This week's exercises concern two-body motion and Kepler's laws. Please submit your solutions through the Blackboard site by 3pm on **Friday 27th November**.

Reading

Read about two-body motion and Kepler's laws in your favourite textbook. e.g.

Fowles & Cassiday	*Analytical Mechanics* (7th ed.)	sections 6.3-6.10
Chow	*Classical Mechanics* (2nd ed.)	sections 12.4, 12.7-12.8
French & Ebison	*Introduction to Classical Mechanics*	pp. 70-87, 223-247
Kibble & Berkshire	*Classical Mechanics* (5th ed.)	sections 4.1-4.4, 7.1-7.2
Thornton &Marion	*Classical Dynamics* (5th ed.)	chapter 8

1 Two-body problem (5 marks)

Two point masses m_1 and m_2 are at positions given by the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. Give expressions for

a) the vector gravitational force **F** between the two masses,

b) the vector gravitational field **g** at $\mathbf{r}_2$ due to m_1;

c) the gravitational potential energy V of the two masses;

d) the gravitational potential Φ at $\mathbf{r}_2$ due to m_1, and

e) the reduced mass of the two masses.

2 Kepler's second law (5 marks)

A particle moves under the influence of a central potential $V(r)$. Show that Kepler's second law – that the radius vector from the force centre to the particle sweeps out area at a constant rate – is true, *whatever the form of V* as long as it is *central*. Derive the law in the form

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{1}{2} r^2 \dot{\vartheta} = \text{constant}$$

where (r,ϑ) are polar coordinates in the orbital plane with the origin at the force centre.

3 Cometary orbit (5 marks)

The minimum distance of a comet from the Sun is observed to be an eighth of the radius of the Earth's orbit (which is assumed to be circular), and its speed at that point (the *perihelion*) is four times the orbital speed v_E of the Earth. The orbits of the Earth and comet are coplanar.

a) Find the comet's speed in terms of v_E when it crosses the Earth's orbit, and the angle at which the orbits cross.

b) Will the comet subsequently escape from the Solar System (i.e. never return)? Explain your reasoning.

4 Spider[1] to Mars (5 marks)

A Tesla roadster is initially in a circular orbit of the Sun at the Earth's orbital radius. A rocket gives it a brief impulse parallel to its velocity, which puts it into a new orbit with aphelion (furthest from the Sun) approaching the asteroid belt, at a distance from the Sun or around 2.6 times the Earth's orbital radius.

a) What is the ratio of the spacecraft's speeds just after and just before the rocket thrust?

b) Use Kepler's third law to determine how long, after the rocket is fired, the roadster reaches the aphelion.

[Assume the orbital period of the Earth to be 1 year, and neglect any gravitational attraction between the spacecraft and the planets.]

[1] http://autoweek.com/article/car-life/convertible-spider-or-spyder-and-what-heck-spider-anyway