Classical Mechanics - exercise sheet 9

This week's exercises address Coriolis forces and rotating coordinate frames. Please submit your solutions through the Blackboard site by 3pm on **Friday 11th December**.

Reading

Revise rotating frames and centrifugal and Coriolis forces in your favourite textbook. e.g.

Fowles & Cassiday	*Analytical Mechanics* (7th ed.)	chapter 5
Chow	*Classical Mechanics* (2nd ed.)	chapter 10
French & Ebison	*Introduction to Classical Mechanics*	chapter 8
Kibble & Berkshire	*Classical Mechanics* (5th ed.)	chapter 5
Thornton & Marion	*Classical Dynamics* (5th ed.)	chapter 10

1 Motion on a rotating planet (5 marks)

The equation of motion of a body with position **r** measured from a point on or near the Earth's surface is

$$\ddot{\mathbf{r}} = \mathbf{g}^* - 2\boldsymbol{\omega} \times \dot{\mathbf{r}},$$

where the body is subject to an effective local gravitational field **g*** and **ω** is the angular velocity of the Earth's rotation. Suppose that the body is projected from **r** = 0 with velocity **v** at time $t = 0$. Explaining any approximations you make in reaching your answer, show that the subsequent position of the body can be written as

$$\mathbf{r} = \mathbf{v}t + \frac{1}{2}\mathbf{g}^* t^2 - \boldsymbol{\omega} \times \mathbf{v}t^2 - \frac{1}{3}\boldsymbol{\omega} \times \mathbf{g}^* t^3.$$

2 Bullet ballistics (5 marks)

A rifle bullet is fired with an initial speed (*muzzle velocity*) *v* horizontally towards the West. In which direction will the Coriolis force deflect it if the experiment is performed in

a) Sapporo, Japan (43º N),

b) Kampala, Uganda (0º), and

c) Montevideo, Uruguay (35º S)?

3 Turntable antics (5 marks)

One of the ants in the Physics kitchen has ventured into the microwave oven, and has just reached the centre of the turntable when a PhD student, ignorant of the ant's presence, turns the oven on. Having read that the shortest distance between two points is a straight line, the ant gallops at 0.1 m s^{-1} along a straight path towards an ant-sized gap at the edge of the oven door. Unfortunately, as the ant heads outwards from the middle of the turntable, an increasing component of its velocity is needed to counter the sideways motion of the turntable beneath it.

The turntable is 0.3 m in diameter and rotates once every 10 s. Will the ant escape? Explain your answer.

4 Isobarios (5 marks)

If the Earth has radius *R* and rotates with angular velocity ω, show that the apparent weight of an object of mass *m* at latitude *l* is given by

$$m\sqrt{\left(g - \omega^2 R\cos^2\lambda\right)^2 + \left(\omega^2 R \sin\lambda\cos\lambda\right)^2}\,.$$

The first space travellers to reach the planet *Isobarios* land in the polar regions and discover, to their surprise, that the apparent weight of objects, measured with a spring balance, does not vary with latitude even though the planet is rotating. The planet is spherically symmetric and the length of the 'day' is 2 hours, 46 minutes and 40 seconds. Calculate the mean density of the planet.

What would the explorers experience if they moved towards the planet's equator?

[The gravitational constant is $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}]