SEMESTER 1 EXAMINATION 2010/11

CLASSICAL MECHANICS

Duration: 120 MINS

VERY IMPORTANT NOTE

Section A answers MUST BE in a separate blue answer book. If any blue answer booklets contain work for both Section A and B questions - the latter set of answers WILL NOT BE MARKED.

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. In a system of N particles, the force on the ith particle is written as

$$\mathbf{F}_i = \mathbf{F}_i^{\rm ext} + \sum_{j \neq i} \mathbf{F}_{ij},$$

where $\mathbf{F}_i^{\rm ext}$ is the external force on the ith particle and $\mathbf{F}_{ij}$ is the force of the jth particle on the ith. If the total external force is given by $\mathbf{F}^{\rm ext} = \sum_{i=1}^{N} \mathbf{F}_i^{\rm ext}$, show that

$$\mathbf{F}^{\rm ext} = \frac{d\mathbf{P}}{dt},$$

where $\mathbf{P}$ is the total linear momentum of the system, which you should define in terms of the individual particle masses m_i and positions $\mathbf{r}_i$. [4]

A2. Explain what is meant by the term *precession* in the context of rotational motion about an axis. [3]

A3. Find an expression relating the acceleration g due to gravity at the Earth's surface to Newton's gravitational constant G and the mass and radius of the Earth (assumed spherically symmetric and non-rotating). [3]

A4. An orbiting planet of mass m is subject only to the gravitational attraction of a sun of mass M. Explain why the angular momentum $\mathbf{L}$ is conserved and why this means that the orbit lies in a plane. [5]

A5. Write down an expression for the centrifugal term entering the "apparent gravity" as seen by an observer in the rotating frame of the Earth and estimate the typical size of the deflection that it induces relative to true gravity. (Recall that the Earth's radius is 6400 km.) [5]

Section B

B1. (a) Show that the moment of inertia of a uniform right circular cylinder of radius a and mass m about its central axis is

$$\frac{1}{2}ma^2.$$

[6]

(b) A straight uniform steel rod of circular section is lying across the horizontal flat bed of a pick-up truck. An absent-minded driver climbs into the cab and drives away, imparting a constant acceleration of $0.5g$ to the truck, having forgotten to tie down the rod. Consequently, the rod rolls without slipping along the truck bed, with the rod's axis perpendicular to the acceleration, until it falls off the end of the bed.

If the truck bed has length $5\,\mathrm{m}$, what is the horizontal velocity (with respect to the ground) of the rod when it falls off? [14]

TURN OVER

B2. The radius of Jupiter's orbit is 5.2 times that of the Earth's orbit (assume the orbits are circular and coplanar). An interplanetary probe is to travel from the Earth to Jupiter along an elliptical orbit which just touches each of the planetary orbits. Find the ratio of the probe's speed just after launching to the Earth's orbital speed. [16]

What will be the fate of the probe if its initial speed is 10% larger? [4]

[Ignore the gravitational attraction of the Earth, Jupiter and any other planets on the probe.]

B3. (a) The equation of motion of a particle moving under gravity with position $\mathbf{x}$ measured from a point on or near the Earth's surface is

$$\ddot{\mathbf{x}} = \mathbf{g}^* - 2\,\boldsymbol{\omega} \times \dot{\mathbf{x}}$$

where $\mathbf{g}^*$ is the effective local gravitational field and $\boldsymbol{\omega}$ is the angular velocity of the Earth's rotation. Suppose the particle is projected from $\mathbf{x} = \mathbf{a}$ with velocity $\mathbf{v}$. Show that,

$$\dot{\mathbf{x}} = \mathbf{v} + \mathbf{g}^* t - 2\,\boldsymbol{\omega} \times (\mathbf{x} - \mathbf{a})$$

in the subseqent motion. If terms of order ω^2 can be neglected, show that,

$$\mathbf{x} = \mathbf{a} + \mathbf{v}t + \frac{1}{2}\,\mathbf{g}^* t^2 - \frac{1}{3}\,\boldsymbol{\omega} \times \mathbf{g}^* t^3 - \boldsymbol{\omega} \times \mathbf{v}\,t^2.$$

[10]

(b) A particle is dropped from height h above the Earth's surface in latitude λ. If all terms of order ω^2 can be neglected, show that the horizontal deflection of the particle when it hits the ground is

$$\frac{1}{3}\,\omega g \left(\frac{2h}{g}\right)^{3/2} \cos\lambda.$$

[8]

(c) In what direction is this deflection ? [2]

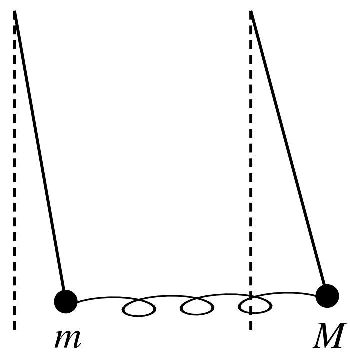
[Neglect air resistance in this question.]

B4. (a) Explain what is meant by a *normal mode* for an oscillating system. [2]

(b) A pair of simple pendulums of equal length ℓ have bobs of mass m and M. The pendulums are coupled by a weak spring of spring constant k, as shown in the diagram below. Consider the normal modes of this system for small oscillations in the plane of the diagram and find

(i) the corresponding eigenfrequencies; [8]

(ii) the corresponding eigenvectors. [4]



(c) The system is released from rest with m in its equilibrium position and M displaced a small distance a directly away from the other pendulum. Find an expression describing the subsequent motion. [6]

END OF PAPER