Preface

These notes accompany the second year core physics course PHYS2006 Classical Mechanics. They are not necessarily complete and are *not* a substitute for the lectures. Certain sections are labelled with a star at the end of the section name, and contain material which is either revision or goes beyond the main line of development of the course. You do not need to consider such sections as part of the syllabus, although they may prove helpful conceptually and useful for other courses.

Background Information

The course continues the mechanics begun in *Motion and Relativity*, PHYS1015. It relates to other physics courses, especially *Quantum Mechanics* and *Crystalline Solids*, and paves the way for *Theories of Matter, Space and Time*.

The course begins by addressing some familiar situations, and progresses to others whose characteristics might be less familiar and perhaps surprising. Throughout, the aim is to establish the formal mathematical basis for classical Newtonian mechanics, and to derive the rules of quite complex behaviours from a few straightforward principles: Newton's laws, the principle of energy conservation, and the properties of Euclidean space. We shall examine the rules governing rigid bodies, their trajectories in external potentials, and what happens when the motions of different bodies are coupled. Along the way, we shall consider planetary motions, gyroscopes and angular momentum, weather systems, and the unique sound of the pianoforte. We shall see the importance of identifying symmetries to help our physical understanding, and glimpse the Hamiltonian and Lagrangian approaches to dynamical situations that link classical mechanics to quantum mechanics and wave physics.

We return to gravity, and derive the important result that the gravitational effect of a spherically symmetric object is the same as the effect of a point-like object, of the same total mass, at its centre. We then discuss Kepler's laws of planetary motion – an early triumph for Newtonian mechanics: to link the observed effects of gravity on the Earth with the force governing celestial motion was a stunning achievement.

We actually begin, however, by considering the motion of systems of particles, allowing us to study problems such as rocket motion. We then look at rotational dynamics, applying Newton's Laws to angular motion, encountering angular velocity, angular momentum and, for systems of particles, the moment of inertia. We shall see some of the seemingly counterintuitive effects that arise in the motion of spinning objects.

While we normally use inertial coordinate systems, the rotation of the Earth on its axis makes coordinate systems fixed to the Earth non-inertial. We shall derive the equation of motion in such a reference frame and see the effects that arise, discussing especially the Coriolis term.

Finally, we consider oscillations and waves in systems of coupled oscillators.

Course Information

Prerequisites The course will assume familiarity with the first year physics and mathematics core courses, particularly PHYS1013, PHYS1015, MATH1006/8 and MATH1007.

Teaching Staff Dr Tim Freegarde is the course coordinator and lecturer. His office is Room 5019 in the School of Physics and Astronomy (building 46) and he can be contacted by email to timf@soton.ac.uk or by telephone on extension 22347.

Course Structure The course comprises about 30 lectures, three per week. Each week there is a one hour workshop where you work on a problem set. Your solutions should be handed in by 2pm on Fridays to the box opposite the First Year Laboratories, and will be returned after marking at the next problems workshop.

Course Materials A handout of printed notes is available (a copy is provided for every student at the start of the course). These notes are *not* necessarily complete, however. Lecture slides, weekly exercises, past exam papers and other course materials may be found on the course website

http://phyweb.phys.soton.ac.uk/quantum/phys2006.php

Study Requirements and Assessment Since it is part of your physics foundation, this course's orientation is towards problem solving, based on a small number of principles. It is very important that you study the weekly problem sheets. They count for 10% of the marks for the course.

The examination will contain two sections, section A with a number of short questions (typically five) all of which must be answered, and section B with four questions from which you must answer two and only two. Section A carries 1/3 and section B carries 2/3 of the examination marks. The way the final mark for this module is worked out is explained in the Student Handbook.

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Reading List and Syllabus

Reading List

D Acheson, From Calculus to Chaos: an Introduction to Dynamics, Oxford University Press 1997

TL Chow, Classical Mechanics, John Wiley 1995

PA Tipler, Physics for Scientists and Engineers (Vol 1, 5th Edition), Freeman 2004

G R Fowles and G I Cassiday, Analytical Mechanics 5th edition, Saunders College Publishing 1993

A P French, Vibrations and Waves, MIT Introductory Physics Series, Van Nostrand Reinhold 1971

A P French and M G Ebison, Introduction to Classical Mechanics, Van Nostrand Reinhold 1986

T W B Kibble, Classical Mechanics 2nd edition, McGraw-Hill 1973

T W B Kibble and F H Berkshire, Classical Mechanics 5th edition, World Scientific Publishing 2004

J B Marion and S T Thornton, Classical Dynamics of Particles and Systems 4th edition, Saunders College Publishing 1995

Fowles and Cassiday's book is full of examples and is the recommended text, although it stops short of discussing one-dimensional crystal models. The treatment of mechanics in Chow's book parallels the course quite closely and has a modern viewpoint. Kibble or Marion and Thornton cover almost everything, but are mathematically more sophisticated. French and Ebison (and French's book on Vibrations and Waves) have good physical explanations but don't cover all the material.

Acheson's book is recommended as supplementary reading and for general background. Although described by its author as "an introduction to some of the more interesting applications of calculus," this book is principally concerned with dynamics, how things evolve in time, and links quite well to some of the topics in this course.

All others are useful to integrate.

Further, two good foundation books to always have at hand are

K F Riley and M P Hobson, Essential Mathematical Methods for the Physical Sciences, Cambridge University Press, 2011

K F Riley and M P Hobson, Foundation Mathematics for the Physical Sciences, Cambridge University Press, 2011

Syllabus

Linear motion of systems of particles

- centre of mass
- total external force equals rate of change of total momentum (internal forces cancel)
- examples (rocket motion, ...)

Angular motion

- rotations, infinitesimal rotations, angular velocity vector
- angular momentum, torque
- angular momentum for a system of particles; internal torques cancel for central internal forces
- rigid bodies, rotation about a fixed axis, moment of inertia, parallel and perpendicular axis theorems, inertia tensor mentioned
- precession (simple treatment: steady precession rate worked out), gyrocompass described

Gravitation and Kepler's Laws

- law of universal gravitation
- gravitational attraction of spherically symmetric objects
- two-body problem, reduced mass, motion relative to centre of mass
- orbits, Kepler's laws
- energy considerations, effective potential

Non-inertial reference frames

- fictitious forces
- motion in a frame rotating about a fixed axis, centrifugal and Coriolis terms — apparent gravity, Coriolis deflection, Foucault's pendulum, weather patterns

Normal modes

- damped and forced harmonic oscillation, resonance (revision)
- coupled oscillators, normal modes
- boundary conditions and eigenfrequencies
- beads on a string

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