SEMESTER 1 EXAMINATION 2007/08

WAVE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. Explain what is meant by (a) a *transverse* wave motion and (b) a *longitudinal* wave motion, and give an example of each. [3]

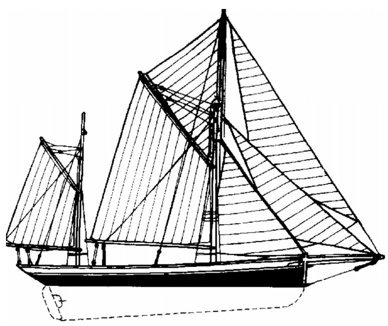
Give an example of a wave which is neither transverse nor longitudinal. [1]

A2. State Fermat's principle of least time for wave propagation. [1]

Draw a diagram showing several possible paths from a source A in air to a point B in a plane-faced block of glass. Explain why the ray path does not in general form a straight line between the two points, and indicate qualitatively which of the paths will be taken. [3]

A3. The historic sailing vessel *'Leader'* is 20.8 m long at her waterline. She will oscillate in a 'pitching' motion (*i.e.* about a horizontal, sideways axis, bow rising while the stern falls and vice-versa) with a period of 4 s. The pitching motion may be excited at sea if she encounters waves at her resonant frequency, and the effect will be largest when the wavelength of such waves is around two boat lengths. Assuming the speed of ocean waves to be given by $v = \sqrt{gh}$, where g is the acceleration due to gravity and h the depth of the water, find the depth of water in which the pitching motion will be most readily excited (and hence the motion will be most uncomfortable when at anchor). [3]

How will this 'most uncomfortable depth' vary if the vessel is moving through the water, for example with a speed v_L and on a course at an angle θ to the direction in which the waves are travelling? [1]



A4. Describe briefly the principles of *Fourier synthesis* and *Fourier analysis*. [2]

Outline how and under which conditions they may be used. [2]

A5. A linear, non-dispersive medium supports travelling waves of the form $y(x,t) = y(\phi)$, where $\phi = ax - bt$ describes the dependence upon position x and time t, the constants a and b being determined by the physical properties of the medium. By considering the path followed by a point of constant wave displacement $y(x,t) = c$, find the propagation speed of the wave. [2]

Although dispersive media do not support such general solutions, they can support travelling waves which are sinusoidal, *i.e.*, $y(\phi) = A\sin(\phi+\phi_0)$, where A and ϕ_0 are constants. By writing the phase ϕ in terms of the angular frequency ω and wavenumber k, show that the *phase velocity* v_p of a point of constant phase ϕ is given by $v_\mathrm{p} = \omega/k$. [1]

Explain briefly the significance of the *group velocity*, $v_\mathrm{g} = \mathrm{d}\omega/\mathrm{d}k$. [1]

Section B

B1. Describe the characteristics of *travelling* and *standing waves*, the differences between them and how they are related. [3]

Waves on a thin, flexible string of mass per unit length ρ and subject to a tension T are governed by the wave equation

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\rho}\frac{\partial^2 y}{\partial x^2}$$

where the wave displacement at time t and position x is $y(x,t)$. Show that travelling waves of the form

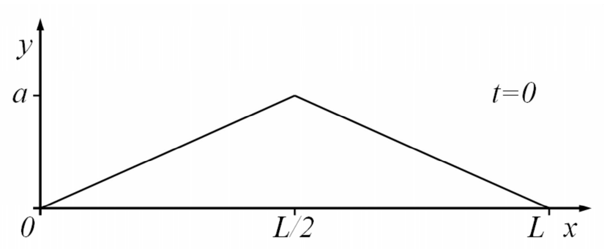
$$y(x,t) = f(u)\,,$$

where $u = x - vt$, may be solutions to this wave equation, and find the two values of v which make them so. [3]

A general solution may be formed from an arbitrary superposition of forward and backward travelling waves

$$y(x,t) = y_+(x - v_+t) + y_-(x - v_-t)$$

where $v_+ = -v_-$. The string of a harp is plucked by pulling it from its midpoint to form the profile shown below. At time $t = 0$, the string is released from rest.



(a) State the specific initial conditions, at time $t = 0$, which define the subsequent motion of the harp string. [2]

(b) Hence determine relationships between the forward and backward travelling components $y_\pm(x,t)$ and the initial profile $y(x,0)$. [4]

(c) Thus determine, and illustrate with a sketch, the string displacement at time $t = L/8v_+$, in the range $L/8 \le x \le 7L/8$. [2]

By considering the *boundary conditions* which apply where the string meets the frame of the harp at $x = 0$ and $x = L$, show that the forward and backward travelling components $y_\pm$ are initially antisymmetric about $x = 0$. Show also that they are periodic, and find the distance after which they each repeat. [3]

Hence sketch the travelling wave components $y_\pm(x, 0)$ at time $t = 0$ over the range $-L \leq x \leq 2L$, and complete your sketch of the string at time $t = (L/8v_+)$.

[3]

B2. Sketch and explain the operation of the *Michelson interferometer*. [5]

The amplitude of the light transmitted by a Michelson interferometer may be determined by summing the amplitudes resulting from the two routes through the interferometer. If the partially-reflecting beamsplitter divides incident light equally between the two paths, the difference in path length between the two routes at normal incidence is s, and the rays of wavelength λ make an angle θ to the mirror normals, write complex exponential expressions for the relative amplitudes of these two contributions. [4]

Hence show that the overall intensity transmitted by the interferometer is given by

$$I_t \propto \cos^2\left(\frac{k\,s}{2}\cos\theta\right)$$

where I_t is the transmitted intensity and $k = 2\pi/\lambda$. [2]

A Michelson interferometer is used to investigate the spectra of a number of light sources, by recording the on-axis transmitted intensity I_t as a function of the path difference s. Sketch, and label with as much detail as you can, the *interferograms* (recordings of I_t vs s) that would be obtained when the light source is

(a) a single-frequency He-Ne laser with a wavelength of 632.8 nm. [3]

(b) a low pressure sodium lamp, which principally emits two wavelengths of 589.0 and 589.6 nm, assuming them to be equal in intensity. [3]

How is the transmission of the interferometer modified if the amplitude transmission t and reflectivity r of the partially-reflecting beamsplitter are not equal? What happens to the fraction of light that is not transmitted by the instrument? [3]

B3. Explain what is meant by *Fraunhofer diffraction*, and give an example of its rôle or occurrence. [4]

Show from first principles that $p(\theta)$, the dependence upon angle of the relative amplitude of light of wavelength λ, when diffracted by a slit of width a, is given by the sinc function

$$p(\theta) \propto \frac{\sin\{\frac{\pi a}{\lambda}\sin\theta\}}{\frac{\pi a}{\lambda}\sin\theta}$$ [4]

Two long, transmitting slits, each of width a, are separated by a non-transmitting region of width $(d - a)$. Show that the Fraunhofer diffraction pattern - *i.e.* the intensity distribution of the light diffracted by the slits at an angle θ - is given by

$$I(\theta) = I_0 \left\{ \frac{\sin\beta}{\beta} \cos\frac{\alpha}{2} \right\}^2$$

when the slits are illuminated by a parallel beam of monochromatic light at normal incidence. Here, $\beta = (1/2)ka\sin\theta$, $\alpha = kd\sin\theta$, $k = 2\pi/\lambda$, and the constant I_0 depends upon the incident intensity. [4]

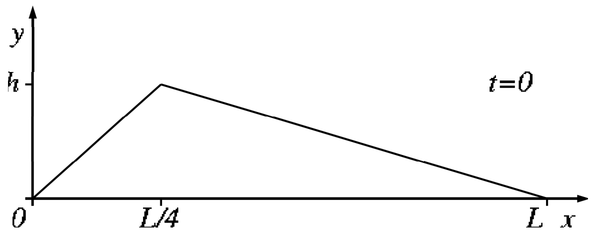
Sketch the intensity distribution when $a = 5\ \mu\mathrm{m}$, $\lambda = 500\ \mathrm{nm}$, and $d \approx 10a$. Your diagram should be labelled to indicate the scale of important features. [4]

Repeat your sketch for the special case $d = a$, and comment on the result. [4]

B4. A guitar string of length L and mass per unit length ρ is subject to a tension T. Sketch the first four harmonics (*i.e.* the fundamental and first three overtones), and determine their frequencies and wavenumbers. [8]

The motion of the string is detected by a pick-up placed beneath the string at $x = d$. Assuming that the performance of the pick-up does not itself depend upon frequency, calculate the relative sensitivities of the pick-up to the first four harmonics of the guitar string (*i.e.* the signal produced by harmonics with the same amplitudes at their anti-nodes) if the string length L is 648 mm and the pick-up is placed at $d = 98$ mm. [4]

The string is plucked by pulling it at a point a quarter of its length from one end, so that it forms the stationary triangle shown below.



Given that the amplitude of the sinusoidal component with wavenumber k in the function $y(x)$ is given by

$$a(k) = \frac{2}{L}\int_0^L y(x)\ \sin kx\ \mathrm{d}x,$$

calculate the amplitudes of the first four sinusoidal components, corresponding to the frequencies calculated above. It may help to use the integral identity

$$\int_0^a x\ \sin x\ \mathrm{d}x = [\sin x - x\cos x]_0^a = \sin a - a\cos a$$ [6]

The plucked string is subsequently released from rest, and its motion is recorded by the pick-up. Calculate the relative strengths of the first four harmonics in the electrical signal from the pick-up. [2]

END OF PAPER