UNIVERSITY OF SOUTHAMPTON PHYS2023W1

SEMESTER 1 EXAMINATION 2008/09

WAVE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A sheet of physical constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

Section A

A1. State Fermat's principle of least time for wave propagation. [1]

Draw a diagram showing several possible paths from a source A in air to a point B in a plane-faced block of glass. Explain why the ray path does not in general form a straight line between the two points, and indicate qualitatively which of the paths will be taken. [3]

A2. Explain briefly the phenomenon of wave *interference*. [1]

Sea waves with straight wavefronts and a period of 1 s strike a straight harbour wall, parallel to the wavefronts, in which there are two small gaps 100 m apart and of equal widths. A fisherman, facing the harbour wall from within the harbour at a point exactly 1000 m from the midpoint between the gaps, notes the amplitude of the transmitted waves as they slap against the hull of his boat, and observes that if he moves to either his left or his right (i.e., in a direction parallel to the harbour wall), the amplitude diminishes, reaching a minimum at a distance of 48 m from his initial position. Sketch this arrangement, indicating the distances involved, and determine the wavelength of the sea waves. [3]

A3. The energy density of a sound wave is, in common with other mechanical wave motions, composed in part by the kinetic energy $\frac{1}{2}\rho(\partial\xi/\partial t)^2$ and in part by the potential energy $\frac{1}{2}E(\partial\xi/\partial x)^2$, where ξ is the longitudinal displacement at position x and time t, ρ is the density of the medium and E its modulus of elasticity. By considering a sinusoidal sound wave of definite frequency, show that these two contributions are equal. [2]

Given that the acoustic intensity is equal to the product of mean energy density and wave speed, find the amplitude of displacement for sound waves in sea water that correspond to the limit of dolphin hearing, $10^{-14}\,\mathrm{W\,m^{-2}}$ at a frequency of 50 kHz. The density of sea water may be taken to be 1025 kg m^{-3}, the modulus of elasticity $E \approx 2.3 \times 10^9$ Pa, and the wave speed is given by $v = \sqrt{E/\rho}$. [2]

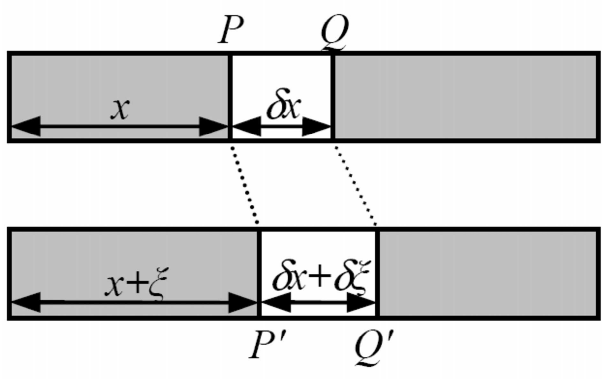
A4. Explain, with examples, the *boundary conditions* that may apply to a wave motion. [2]

By considering the different boundary conditions governing the instruments, explain why guitar and violin strings can support all harmonics of the fundamental frequency, but instruments like the clarinet produce only odd-numbered harmonics. [2]

A5. Why are *sinusoidal* wave motions so often considered? How are they related to *complex exponential* waves? [4]

Section B

B1. The figure below shows the motion of a longitudinal volume element of an elastic medium which is displaced (bottom) by $\xi(x,t)$ from its rest position (top).



By considering the pressure within, and net force acting upon, the longitudinal element of density ρ and elasticity E, show that the wave equation governing its longitudinal motion will be

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 \xi}{\partial x^2}\,. \qquad [5]$$

Hence estimate the speed of sound in stainless steel, assuming a density of 8030 kg m^{-3} and elasticity of 1.93×10^{11} Pa. [2]

The continuity conditions for the longitudinal displacement ξ of sound waves as they pass, at $x = x_0$, from a region A of density ρ_A and elasticity E_A into another region B of density ρ_B and elasticity E_B are

$$\begin{aligned} \xi_A(x_0,t) &= \xi_B(x_0,t) \\ E_A\frac{\partial \xi_A}{\partial x}(x_0,t) &= E_B\frac{\partial \xi_B}{\partial x}(x_0,t)\,. \end{aligned}$$

State the physical reason for each of these conditions. [2]

The displacements of travelling sinusoidal sound waves in the two regions A and B may be written in the form

$$\xi_{A,B}(x,t) = a_{A,B}\cos\left(\omega t - k_{A,B}(x - x_0)\right)$$

For a given angular frequency ω, state the possible values for the wavenumber $k_{A,B}$ in the two regions. [2]

Hence show that when a sound wave of amplitude a_i passes from region A to region B it results in a reflected wave of amplitude a_r where

$$\frac{a_r}{a_i} = \frac{Z_A - Z_B}{Z_A + Z_B}$$

and the acoustic impedance Z is given in terms of the density ρ and elasticity E by $Z = \sqrt{E\rho}$. [5]

Ultrasound imaging depends upon the reflection of high frequency sound at the interfaces between different media. Given the data in the table below, and assuming human tissue to be approximately homogeneous in its acoustic properties, find the fraction of the wave *intensity* that is reflected when ultrasound is normally incident upon the interface between tissue and amniotic fluid. [2]

material	density (kg m^{-3})	elasticity (Pa)
tissue	1070	2.7×10^9
amniotic fluid	1000	1.5×10^9

What other considerations would determine the overall strength of the ultrasound signal obtained when imaging a human foetus within the uterus? [2]

B2. Sketch and explain the operation of the *Michelson interferometer*. [5]

The amplitude of the light transmitted by a Michelson interferometer may be determined by summing the amplitudes resulting from the two routes through the interferometer. If the partially-reflecting beamsplitter divides incident light equally between the two paths, the difference in path length between the two routes at normal incidence is s, and the rays of wavelength λ make an angle θ to the mirror normals, write complex exponential expressions for the relative amplitudes of these two contributions. [4]

Hence show that the overall intensity transmitted by the interferometer is given by

$$I_\mathrm{t} \propto \cos^2\left(\frac{k\,s}{2}\cos\theta\right)$$

where I_t is the transmitted intensity and $k = 2\pi/\lambda$. [2]

A Michelson interferometer is used to investigate the spectra of a number of light sources, by recording the on-axis transmitted intensity I_t as a function of the path difference s. Sketch, and label with as much detail as you can, the *interferograms* (recordings of I_t vs s) that would be obtained when the light source is

(a) a single-frequency argon ion laser with a wavelength of 488 nm. [3]

(b) a low pressure rubidium lamp, which principally emits two wavelengths of 780.2 and 795.0 nm, assuming them to be equal in intensity. [3]

How is the transmission of the interferometer modified if the amplitude transmission t and reflectivity r of the partially-reflecting beamsplitter are not equal? What happens to the fraction of light that is not transmitted by the instrument? [3]

B3. Explain what is meant by *dispersion*, and by the *phase* and *group velocities*. Give expressions for the phase and group velocities in terms of the angular frequency ω and wavenumber k of a wave. [6]

The deep *swell* of low frequency ocean waves may be modelled by assuming that 'slices' of the ocean move horizontally, the small vertical motion of the surface resulting from the horizontal contraction of a slice and the motion of a slice resulting from an imbalance in hydrostatic pressure across it. The application of Newtonian mechanics to this model produces two equations relating the wave height $h(x,t)$ and the horizontal velocity of the slice $v(x,t)$ at any position x and time t:

$$\begin{aligned}\frac{\partial h}{\partial t} &= -h_0\frac{\partial v}{\partial x}\\ \frac{\partial v}{\partial t} &= -g\frac{\partial h}{\partial x}\end{aligned}$$

where h_0 is the (undisturbed) depth of the ocean and g the acceleration due to gravity. Show that this pair of partial differential equations may be combined to derive the wave equation for deep ocean waves,

$$\frac{\partial^2 h}{\partial t^2} = g\,h_0\frac{\partial^2 h}{\partial x^2}$$

and deduce the phase and group velocities for deep water waves. [5]

Waves of higher frequency, in contrast, affect only the upper parts of the ocean, and are governed by modified versions of the wave equation. The result is that for shallow water waves, with wavelengths much less than the water depth, the dispersion relation becomes

$$\omega = \sqrt{g\,k}$$

Show that the phase velocity for shallow water waves of angular frequency ω is given by $v_p(\omega) = \sqrt{g/k}$, derive the corresponding *group velocity*, and comment upon your results. [4]

The disturbance of a boat moving through water with constant speed v_{boat} will reinforce wave components of angular frequency ω that travel at an angle θ to the boat if

$$v_p(\omega) = v_{\text{boat}} \cos\theta$$

Derive an expression for the wavelength of reinforced waves as a function of θ and v_{boat}, and show that the component travelling in the same direction as the boat will have a wavelength $\lambda = 2\pi v_{\text{boat}}^2/g$. [3]

If the *hull speed* (above which the drag increases sharply) is that for which the wavelength is equal to the boat length, calculate the hull speed for a racing rowing boat of length 17.3 m. [2]

B4. What is meant by the *Fourier transform*? How may it be defined mathematically? [4]

A pulsed radar at an airfield emits microwave bursts of duration T of a single frequency $f_0 = \omega_0/2\pi$ (where $\omega_0 T \gg 1$), so that the wave amplitude is

$$a(t) = \begin{cases} \cos\omega_0 t & (-T/2 \le t \le T/2) \\ 0 & (t < -T/2,\ t > T/2) \end{cases}$$

Show that the Fourier transform of a single burst is given by

$$b(\omega) \propto \frac{\sin\{(\omega_0 - \omega)T/2\}}{\omega_0 - \omega} + \frac{\sin\{(\omega_0 + \omega)T/2\}}{\omega_0 + \omega}$$

and sketch the amplitude and intensity spectra corresponding to your result for frequencies around ω_0, where the second term may be neglected. [8]

Given that $\{\sin(a)/a\}^2 = 1/2$ when $a = 1.392$, derive the full width at half maximum intensity of the spectrum of each burst. [4]

The radar, which operates by reflecting microwave bursts from airborne aircraft, is required to measure aircraft positions with a precision of 15 m. Taking the speed of electromagnetic waves to be 3×10^8 m s^{-1}, estimate the maximum allowable duration of the radar pulse, and hence estimate the minimum bandwidth of the transducers used to generate it. [4]

END OF PAPER