
SEMESTER 1 EXAMINATION 2015-2016

WAVE PHYSICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language Word to Word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

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Section A

A1. Mountain lee waves are transverse (vertical) motions of the atmosphere when a strong wind blows over a mountain or hill; they occur downwind of the mountain, extend to all altitudes, and are beloved of glider pilots who use them to soar to heights of many kilometres. The vertical motion of the atmospheric airmass can be taken to be roughly sinusoidal,

$$\psi(x,t) = \psi_0 \cos(kx) = \psi_0 \cos(kx' + \omega t),$$

where $\psi(x,t)$ is the vertical displacement of the airmass from its initial altitude, x is its horizontal distance downwind of the mountain, $x' = x - v_{\mathrm{wind}} t$ is its position within the moving airmass as it travels with speed v_{wind} relative to the ground, and $\omega = v_{\mathrm{wind}} k$.

Show that the vertical velocity of the air at position x and time t is given by

$$v_{\mathrm{vert}} = -\psi_0 \, v_{\mathrm{wind}} \, k \sin(kx). \qquad [\,2\,]$$

A glider requires the air to rise with a minimum speed of 1 m s^{-1} for it to remain aloft. If v_{wind} = 10 m s^{-1} and $2\pi/k = 5{,}000\,\mathrm{m}$, find the minimum amplitude ψ_0 of the wave motion. [2]

A2. Describe the Doppler effect, and discuss two methods by which it may be theoretically derived. [4]

A3. Explain why a laser pulse must comprise a range of optical frequencies, and describe how the frequency range is related to the length of the pulse. [4]

A4. The antenna for a stereo radio receiver comprises a metal rod (cut at the middle for the connection to the radio), which acts as a resonator for radio waves of frequency 100 MHz. By considering the boundary conditions at the ends of the rod, and assuming the speed of electromagnetic waves along the rod to equal the speed of light in vacuum, c, calculate the shortest total length of the rod. [3]

At which other frequencies will the antenna also act as a resonator? [1]

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A5. Explain what are meant by the terms *travelling* and *standing* waves. [2]

Show, with an example, how travelling waves may be superposed to form a standing wave, and vice-versa. [2]

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Section B

B1. A coaxial cable comprises an inner conductor of radius a that lies concentrically within a cylindrical outer conductor of radius b. The space between the two conductors is filled with a non-magnetic dielectric of relative permittivity ε. The capacitance and inductance per unit length, C and L, are given by

$$\begin{aligned} C &= \frac{2\pi\varepsilon_0\varepsilon}{\ln(b/a)}, \\ L &= \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right). \end{aligned}$$

(a) Sketch the geometry of the coaxial cable, indicating the voltages $V(x,t)$ and currents $I(x,t)$, charges $Q(x,t)$, and electric and magnetic fields $E(x,t)$ and $B(x,t)$, for an element of length δx at position x and time t. [4]

(b) Derive the relationship

$$C\frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x}.$$ [3]

(c) Derive the relationship

$$L\frac{\partial I}{\partial t} = -\frac{\partial V}{\partial x}.$$ [3]

(d) Hence derive the wave equation

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial x^2},$$ [3]

and give an expression for the phase velocity of electromagnetic waves along the cable. [1]

(e) Given that the diameters of the inner and outer conductors are 1 mm and 5 mm respectively, and the space between them is filled with solid PTFE with a relative permittivity of 2.05, find the capacitance and inductance per unit length. [4]

(f) Hence determine the time taken for a signal to propagate along a 1 m length of the cable. [2]

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B2. (a) Sketch and explain the operation of the *Michelson interferometer*. [5]

(b) The amplitude of the light transmitted by a Michelson interferometer may be determined by summing the amplitudes resulting from the two routes through the interferometer. Given that the partially-reflecting beamsplitter divides incident light equally between the two paths, the difference in path length between the two routes at normal incidence is s, and the rays of wavelength λ make an angle θ to the mirror normals, sketch the geometry and write complex exponential expressions for the relative amplitudes of these two contributions. [4]

(c) Hence show that the overall intensity transmitted by the interferometer is given by

$$I_t \propto \cos^2\left(\frac{k\,s}{2}\cos\theta\right),$$

where I_t is the transmitted intensity and $k = 2\pi/\lambda$. [2]

(d) A Michelson interferometer is used to investigate the spectra of a number of light sources, by recording the on-axis transmitted intensity I_t as a function of the path difference s. Sketch, and label with as much detail as you can, the *interferograms* (recordings of I_t vs s) that would be obtained when the light source is

(i) a single-frequency telecommunications laser with an infrared wavelength of 1561.4 nm. [3]

(ii) a low-pressure potassium lamp, which principally emits two wavelengths of 766.5 nm and 769.9 nm, taking them to be equal in intensity. [3]

(e) How is the transmission of the interferometer modified if the amplitude transmission t and reflectivity r of the partially-reflecting beamsplitter are not equal? What happens to the fraction of light that is not transmitted by the instrument? [3]

TURN OVER

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B3. (a) Explain what is meant by *Fraunhofer diffraction*, and give an example of its rôle or occurrence. [4]

(b) Show from first principles that $p(\vartheta)$, the dependence upon angle of the relative amplitude of light of wavelength λ, when diffracted by a slit of width a, is proportional to the sinc function

$$p(\vartheta) \propto \frac{\sin\{\frac{\pi a}{\lambda}\sin\vartheta\}}{\frac{\pi a}{\lambda}\sin\vartheta}.$$ [4]

(c) Two long, transmitting slits, each of width a, are separated by a non-transmitting region of width $(d - a)$. Show that the Fraunhofer diffraction pattern - *i.e.* the intensity distribution of the light diffracted by the slits at an angle ϑ - is given by

$$I(\vartheta) = I_0 \left\{ \frac{\sin\alpha}{\alpha}\cos\beta \right\}^2$$

when the slits are illuminated by a parallel beam of monochromatic light at normal incidence. Here, $\alpha = (ka/2)\sin\vartheta$, $\beta = (kd/2)\sin\vartheta$, $k = 2\pi/\lambda$, and the constant I_0 depends upon the incident intensity. [4]

(d) Sketch the intensity distribution when $a = 10\ \mu\mathrm{m}$, $\lambda = 1\ \mu\mathrm{m}$, and $d \approx 3a$. Your diagram should be labelled to indicate the scale of important features. [4]

(e) Repeat your sketch for the special case $d = a$, and comment on the result. [4]

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B4. (a) Explain what is meant by *dispersion*. Give examples of practical manifestations of dispersion, and of an application that exploits it. [5]

(b) Show that the wave equation

$$i\,m\frac{\partial\psi}{\partial t} = -\frac{\partial^2\psi}{\partial x^2}$$

where m is a constant, has complex exponential travelling wave solutions of the form

$$\psi(x,t) = \psi_0 \exp\left[i(kx - \omega t)\right].$$

Explain the significance of the parameters k and ω, and show that the *dispersion relation* between k and ω is given by $m\omega = k^2$. [5]

(c) What is meant by the *phase* and *group velocities*? Give, for the above example, an expression for the phase velocity v_p in terms of ω. [3]

(d) A travelling wave has two complex exponential components, equal in magnitude, with frequencies $\omega_0 \pm \delta\omega$ and wavenumbers $k_0 \pm \delta k$. Show that the wave may be written in the form

$$\psi(x,t) = \psi_1 \exp\left[i(k_0 x - \omega_0 t)\right]\ \cos(\delta k\, x - \delta\omega\, t)$$

and thus takes the form of a complex exponential travelling wave that is modulated by a slowly-varying, real periodic function. [4]

(e) By considering how δk depends upon $\delta\omega$, show that the phase velocity of the wave differs from the group velocity of the modulating envelope by a factor of two. [3]

END OF PAPER

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