UNIVERSITY OF Southampton
School of Physics and Astronomy

Wave Physics

PHYS 2023

Tim Freegarde

2020-21

11 Interference

Wave propagation

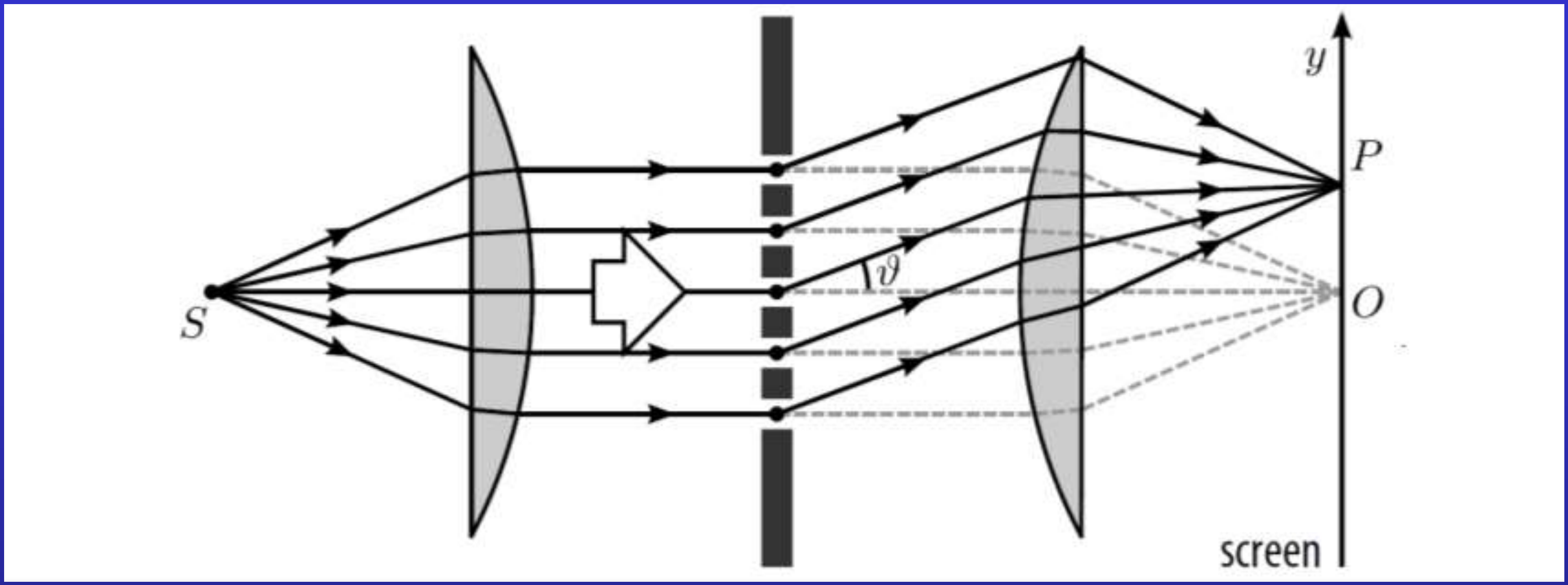
waves are

- collective bulk disturbances, in which
- motion is a delayed response
- to neighbouring motions

when propagation follows multiple routes

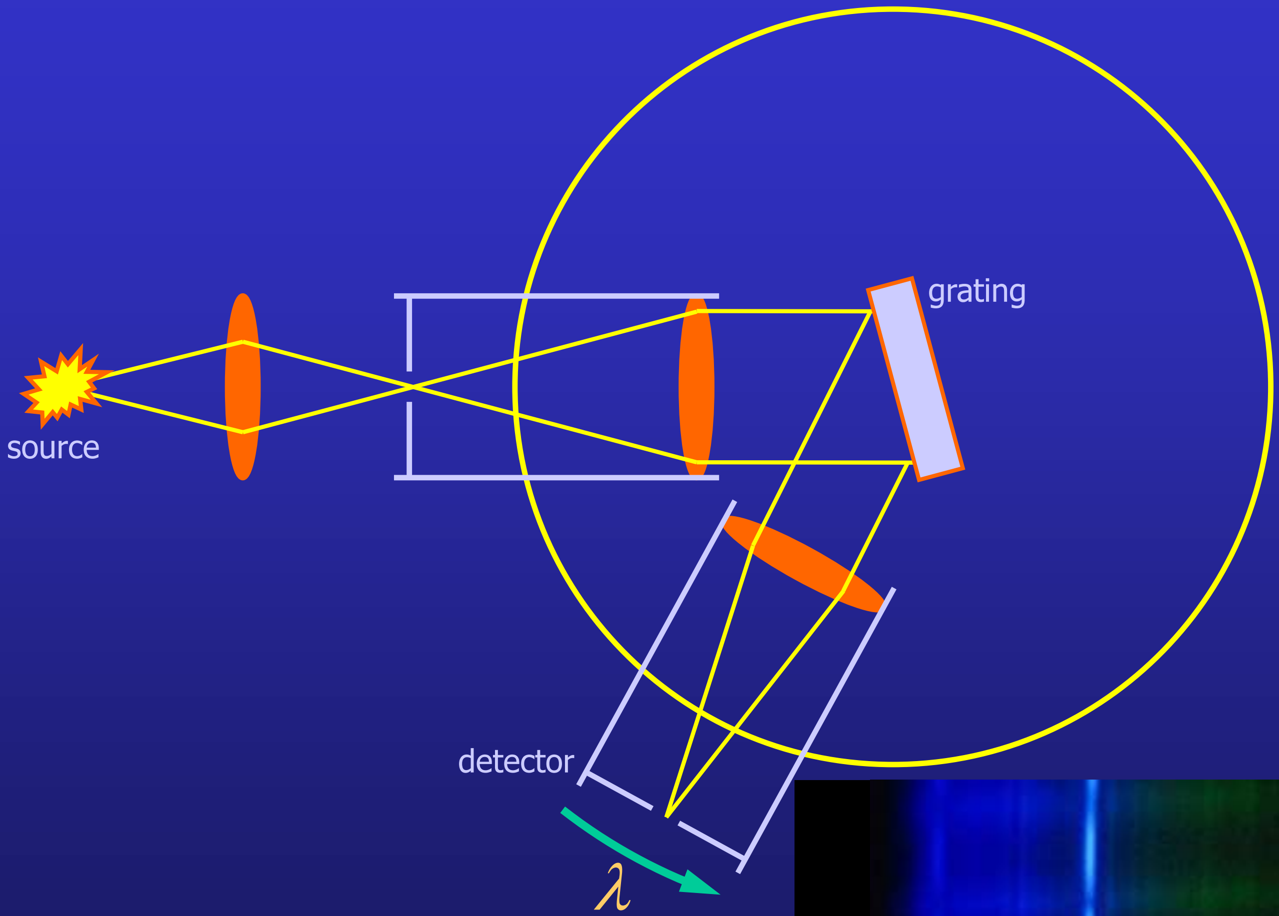
- the amplitudes are added
- waves propagate via all possible routes

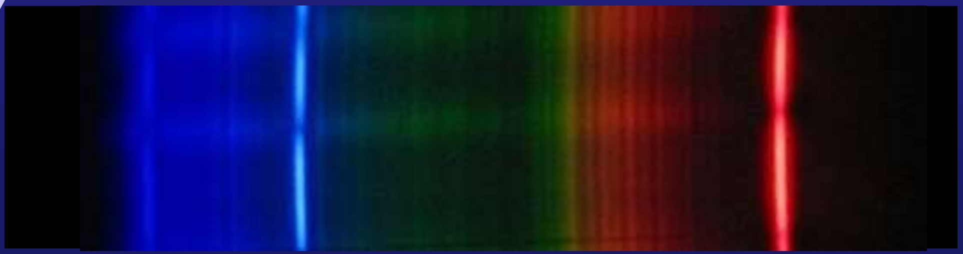
Fraunhofer diffraction



- observed at infinity with plane-wave illumination
- observed using imaging lens and plane-wave illumination
- observed in image plane of source
- path length phase is linear function of transverse coordinate in object

Grating spectrometer





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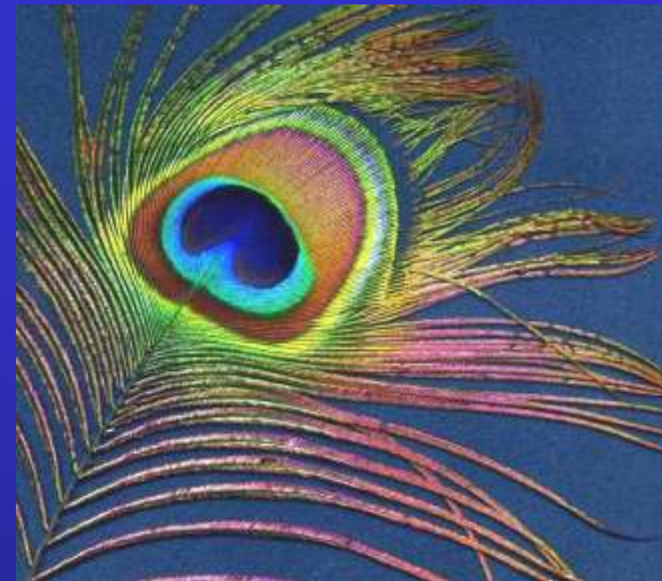
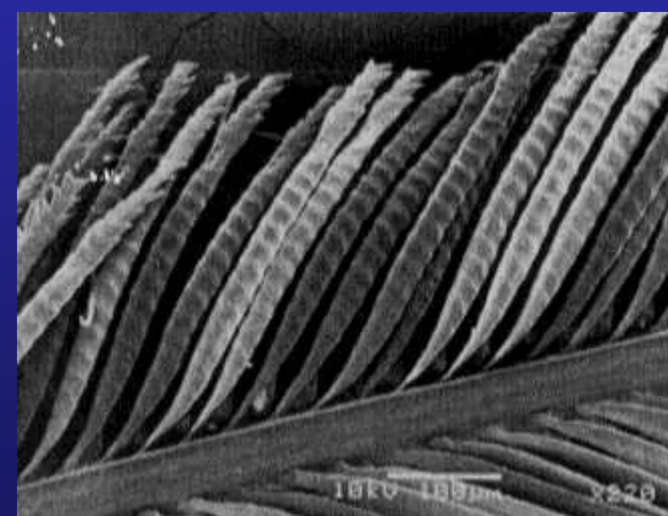
Diffraction

- irridescence of feathers (Grimaldi, 1665)
- interference by division of wavefront

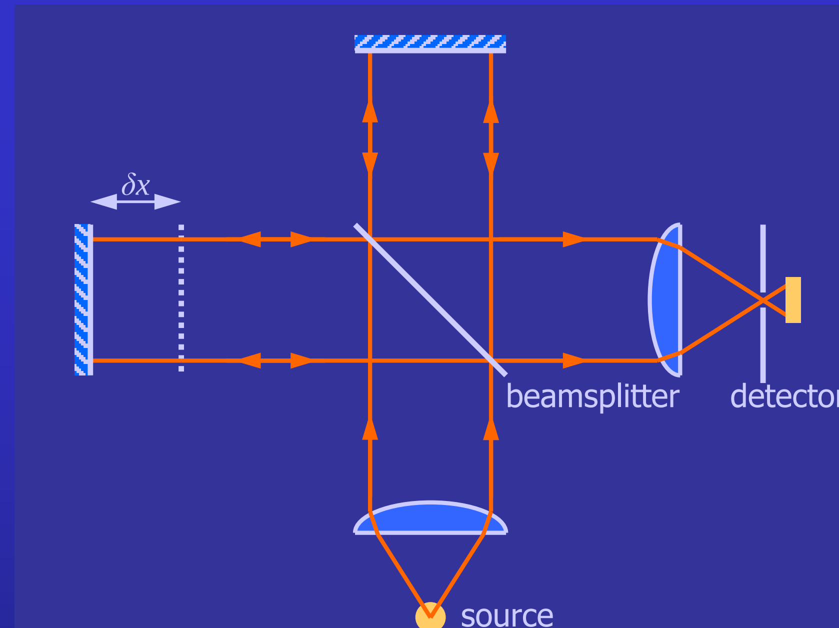




S Yoshioka & S Kinoshita, Forma **17** 169 (2002)

Michelson interferometer

- interference by division of amplitude



Consider a wave propagating from the source that, after the collimating lens, is

$$\underline{E}(x,t) = \underline{E}_0 \cos(kx - \omega t)$$

This is split and sent by two paths a and b so that two components approach the focussing lens:

$$\underline{E}_a(x,t) = rt\underline{E}_0 \cos[k(x_0+2x_a) - \omega t] = rt\underline{E}_0 \cos[k(x_0+x_a+x_b+(x_a-x_b)) - \omega t]$$

$$\underline{E}_b(x,t) = tr\underline{E}_0 \cos[k(x_0+2x_b) - \omega t] = tr\underline{E}_0 \cos[k(x_0+x_a+x_b-(x_a-x_b)) - \omega t]$$

By using the identity $\cos(A+B) = \cos A\cos B - \sin A \sin B$, we write the superposition as

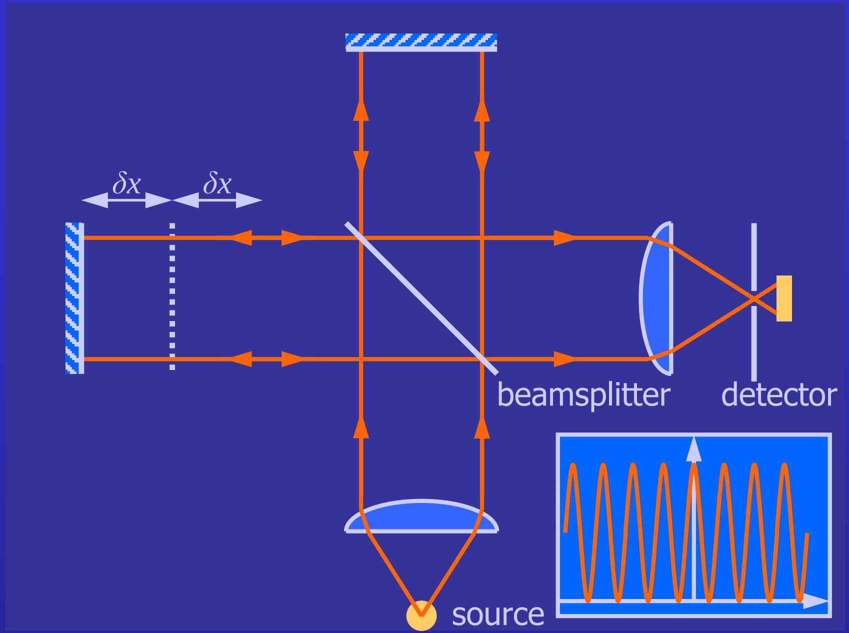
$$\underline{E}_a(x,t) + \underline{E}_b(x,t) = \underbrace{2rt\underline{E}_0 \cos[k(x_0+x_a+x_b) - \omega t]}_{\text{SINGLE BEAM ALONG MEAN PATH}} \underbrace{\cos[k(x_a-x_b)]}_{\text{MODULATION}}$$

The (time-averaged) intensity will be

$$I(\delta x) \propto 4RT \cos^2[k\delta x] \quad \text{where } \delta x \equiv x_a - x_b,\ R \equiv r^2,\ T \equiv t^2$$

Michelson interferometer

- interference by division of amplitude



Consider a wave propagating from the source that, after the collimating lens, is

$$E(x,t) = E_0 \cos(kx - \omega t)$$

This is split and sent by two paths a and b so that two components approach the focussing lens:

$$E_a(x,t) = rtE_0 \cos[k(x_0+2x_a)-\omega t] = rtE_0 \cos[k(x_0+x_a+x_b+(x_a-x_b))-\omega t]$$

$$E_b(x,t) = trE_0 \cos[k(x_0+2x_b)-\omega t] = trE_0 \cos[k(x_0+x_a+x_b-(x_a-x_b))-\omega t]$$

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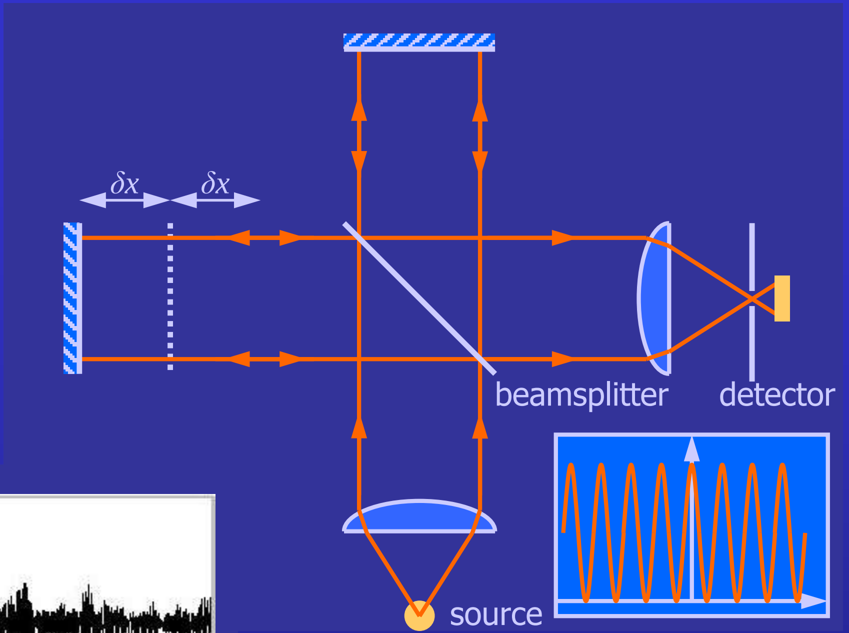
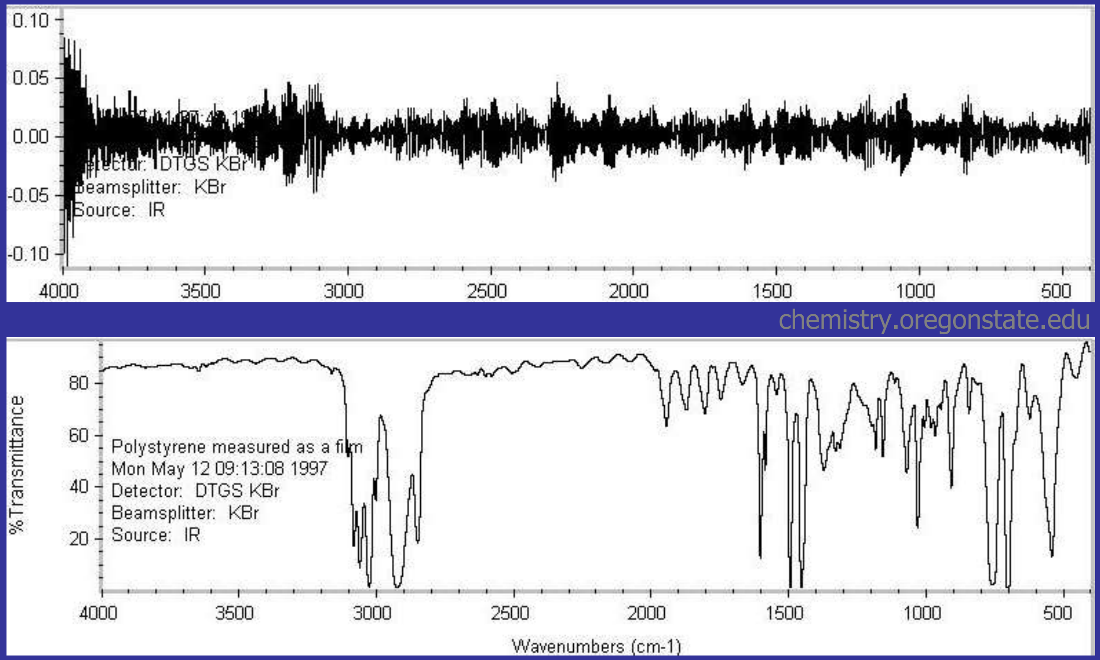
Michelson interferometer

- interference by division of amplitude
- measurement of wavelengths, spectra and lineshapes
- FTIR: Fourier transform infrared





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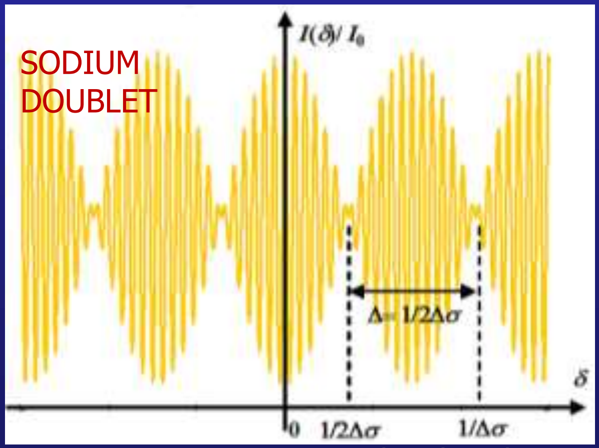

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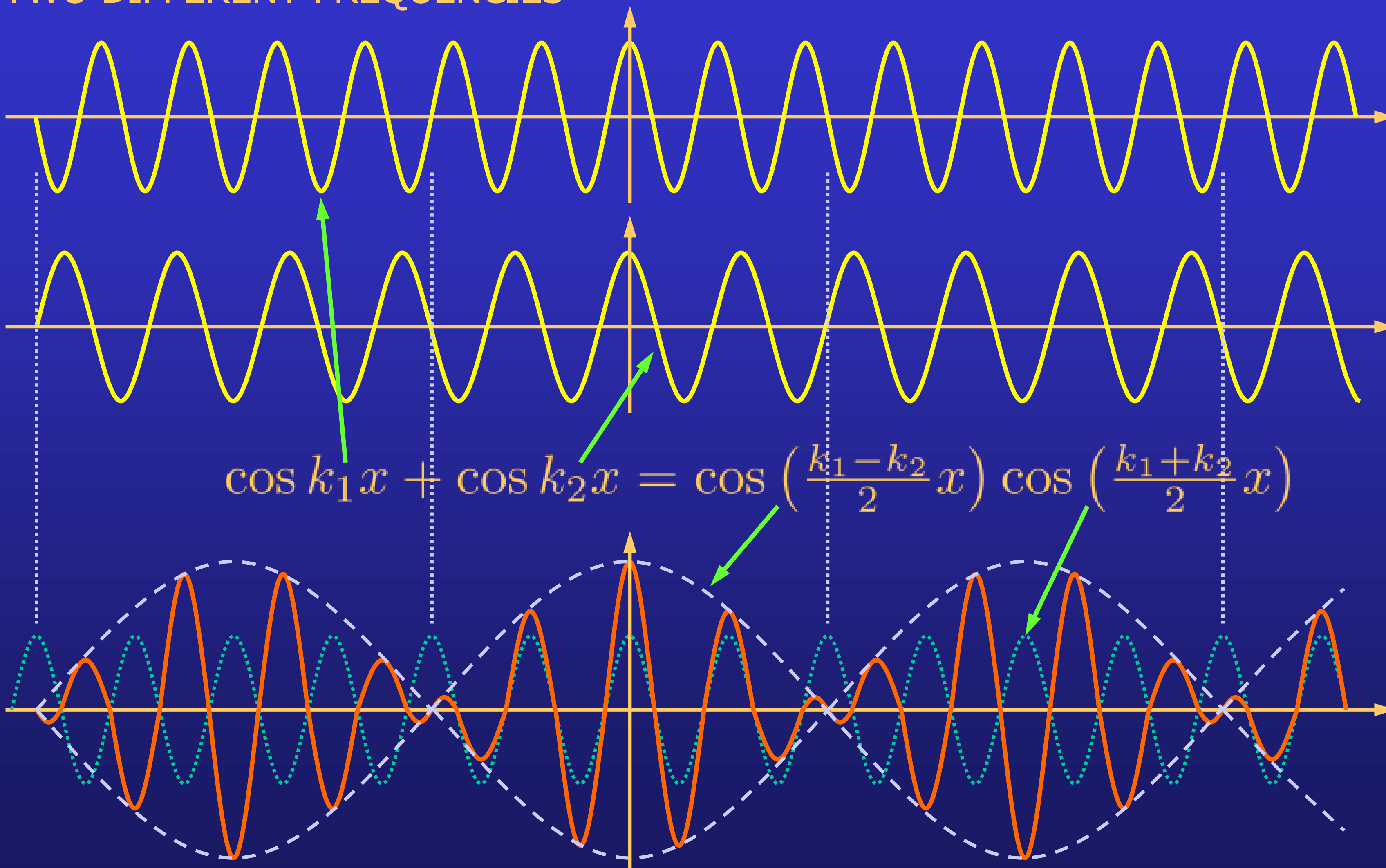
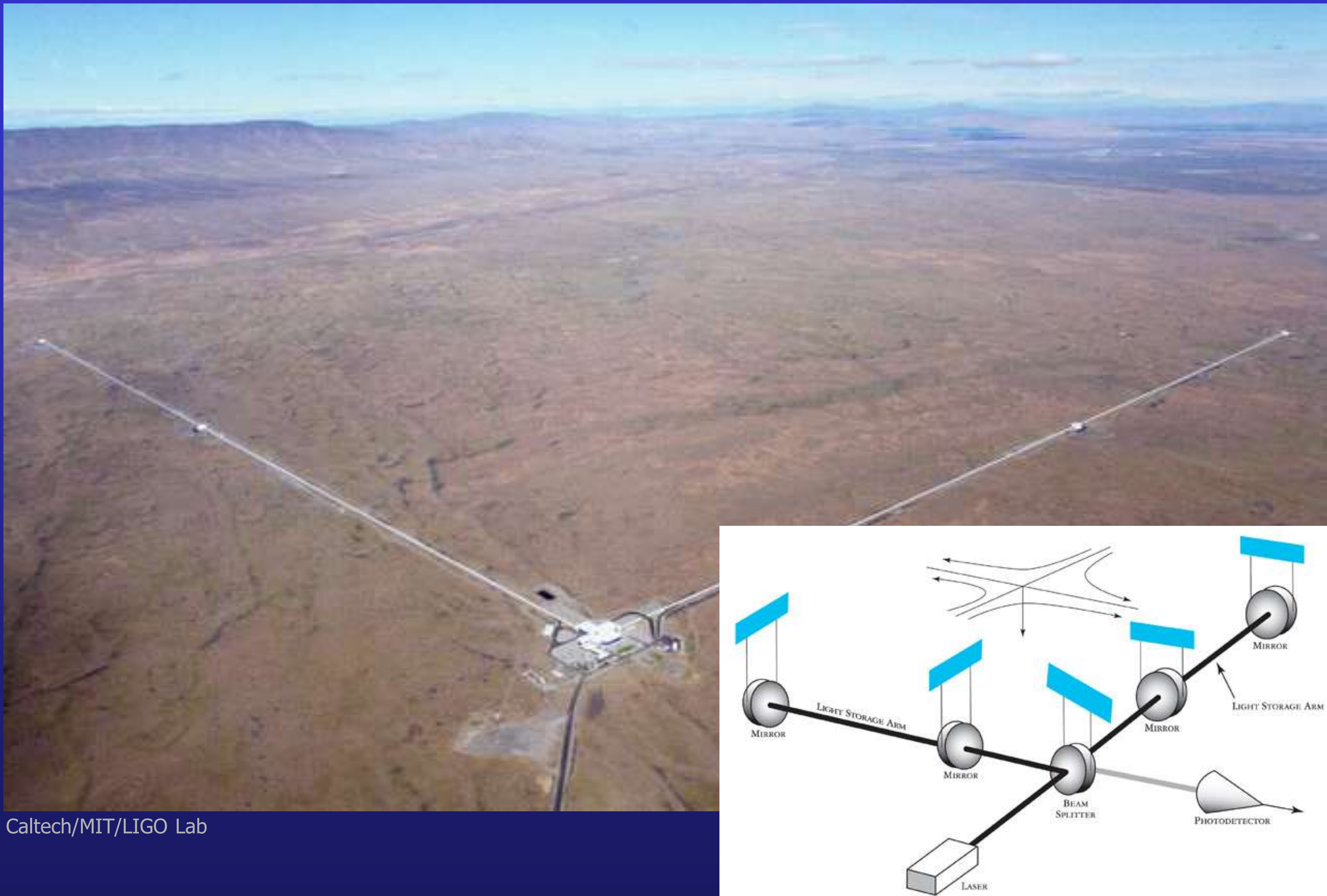
Beating

TWO DIFFERENT FREQUENCIES



$$\cos k_1 x + \cos k_2 x = \cos\left(\frac{k_1 - k_2}{2} x\right) \cos\left(\frac{k_1 + k_2}{2} x\right)$$

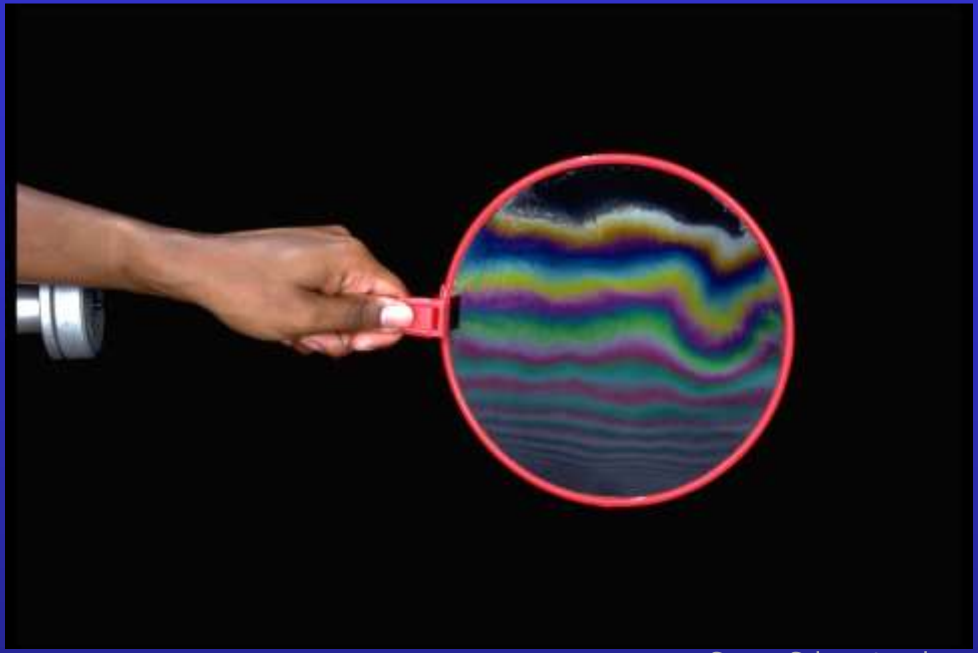
LIGO



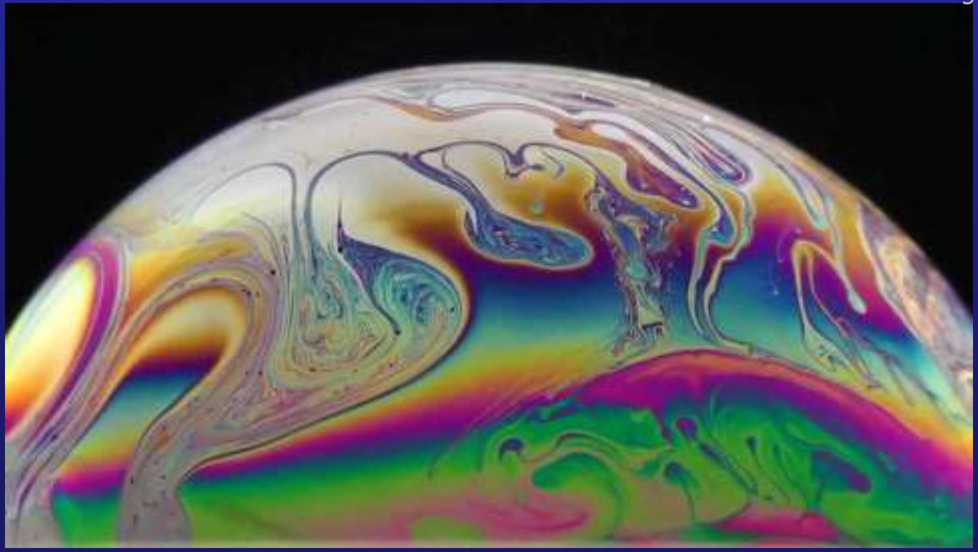
Caltech/MIT/LIGO Lab

Optics & Photonics

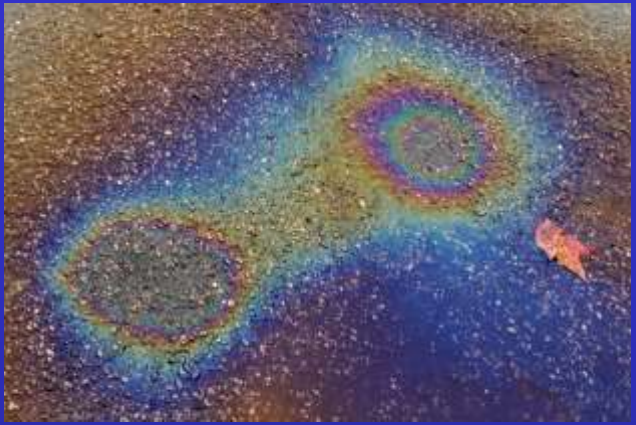
Thin films



Susan Schwartzenberg



Thomas Poersch



Caroline Mockett

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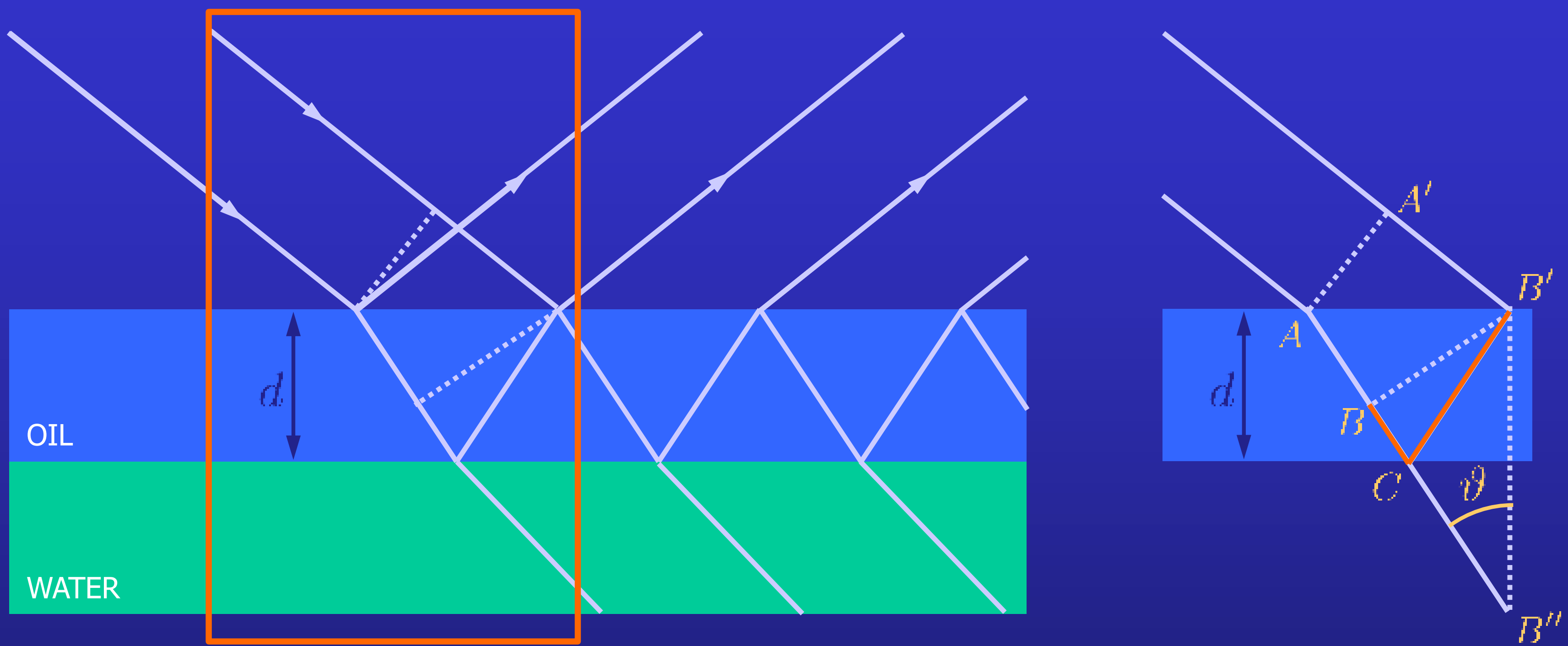
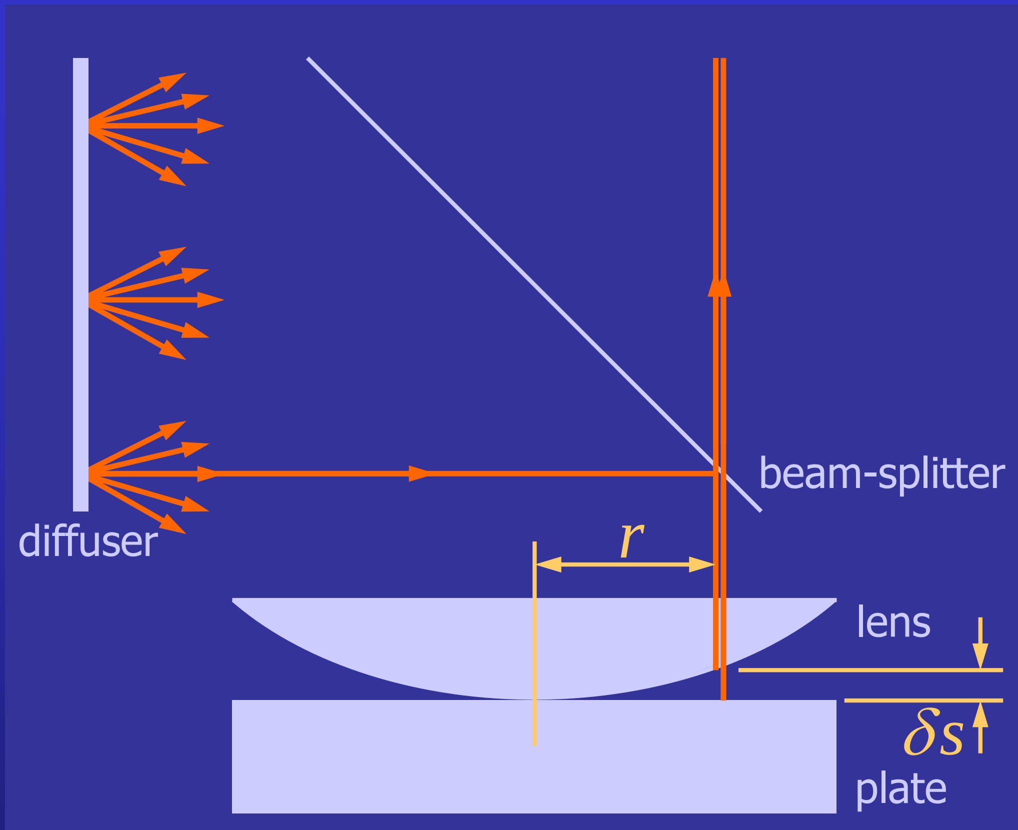
Thin film interference

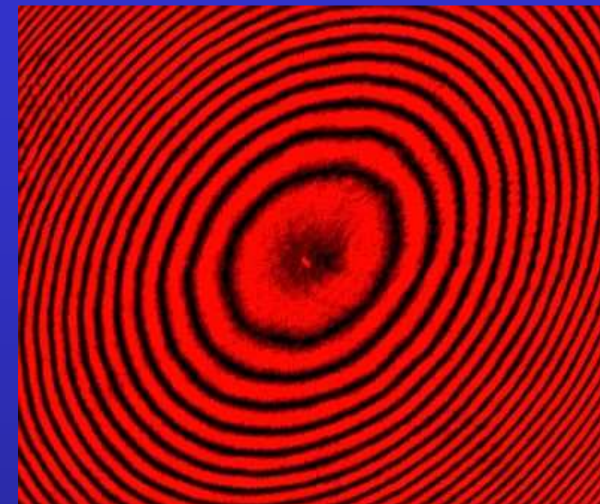


- propagation time $T_{AB} = T_{A'B'}$
- path difference $BCB' = BCB'' = 2d\cos\vartheta$
- for constructive interference $2d\cos\vartheta = n\lambda$

…or destructive interference if phase changes upon reflection

Newton’s rings





- apply to sinusoidal waves by taking into account the phase with which components arrive
- combine by adding the amplitudes
- contributions may therefore interfere constructively or destructively

Newton's rings

Interference occurs between the paths SAS and SBS, which differ in length by distance $2x$.

Pythagoras' theorem gives

$$R^2 = r^2 + (R-x)^2$$

$$= r^2 + R^2 - 2Rx + x^2$$

$$\Rightarrow \quad r^2 = (2R-x)x$$

$$\approx 2Rx \quad \text{for } x \ll R$$

The phase difference between the two paths will be

$$\delta\phi = 2\pi\frac{2x}{\lambda} + \Delta\phi$$

where $\Delta\phi \equiv \phi_1 - \phi_2$ is the difference in phase change upon reflection from the two surfaces.

For one glass-air and one air-glass interface, $\Delta\phi = \pi$.

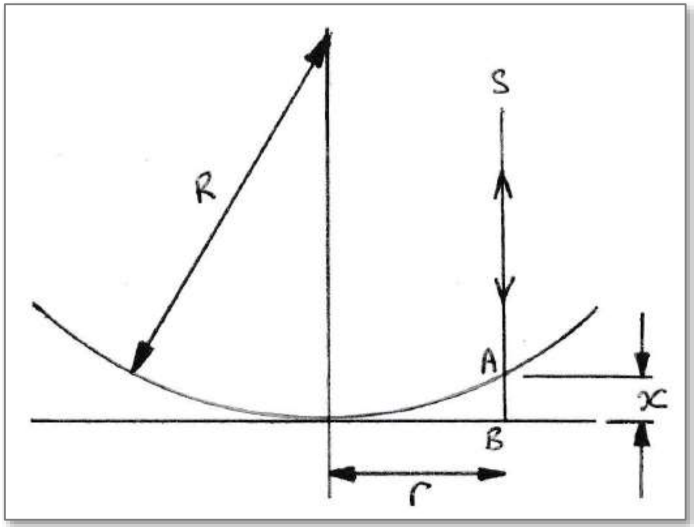
$$\Rightarrow \quad \delta\phi = 4\pi\frac{x}{\lambda} + \pi$$

For a bright fringe, $\delta\phi = 2m\pi$; for a dark fringe, $\delta\phi = (2m+1)\pi$

$\Rightarrow$ dark fringes occur when $4\pi\frac{x}{\lambda} + \pi = (2m+1)\pi$

i.e. $$x = \frac{m}{2}\lambda$$

$$\Rightarrow \quad r = \sqrt{2R\frac{m}{2}\lambda} = \sqrt{mR\lambda}$$



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